Lecture 08: Principles of Parallel Algorithm Design

Concurrent and Multicore Programming
CSE 436/536

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Last lecture: Algorithms and Concurrency

- Introduction to Parallel Algorithms
  - Tasks and Decomposition
  - Processes and Mapping
- Decomposition Techniques
  - Recursive Decomposition (divide-conquer)
  - Data Decomposition (input, output, input+output, intermediate)

- Terms and concepts
  - Task dependency graph, task granularity, degree of concurrency
  - Task interaction graph, critical path
- Examples:
  - Dense vector addition, matrix vector product
  - Dense matrix matrix product
  - Database query
  - Quicksort, MIN

Today's lecture

- ☛ **Decomposition Techniques - continued**
  - Exploratory Decomposition
  - Hybrid Decomposition

Mapping tasks to processes/cores/CPU/PEs

- **Characteristics of Tasks and Interactions**
  - Task Generation, Granularity, and Context
  - Characteristics of Task Interactions
- **Mapping Techniques for Load Balancing**
  - Static and Dynamic Mapping
- **Methods for Minimizing Interaction Overheads**
- **Parallel Algorithm Design Models**

Exploratory Decomposition

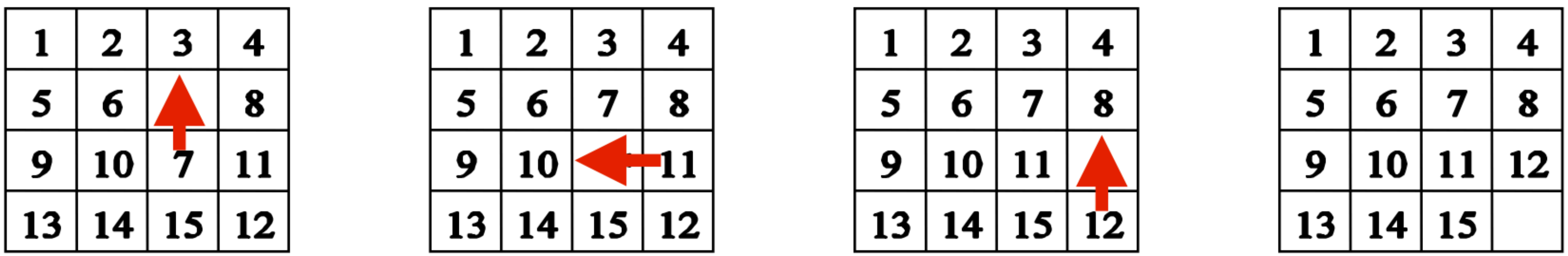
- Decomposition is fixed/static from the design
  - Data and recursive

- Exploration (search) of a state space of solutions
  - problem decomposition reflects shape of execution
  - Goes hand-in-hand with its execution
- Examples
  - discrete optimization, e.g. 0/1 integer programming
  - theorem proving
  - game playing

Exploratory Decomposition: Example

Solve a 15 puzzle

- Sequence of three moves from state (a) to final state (d)



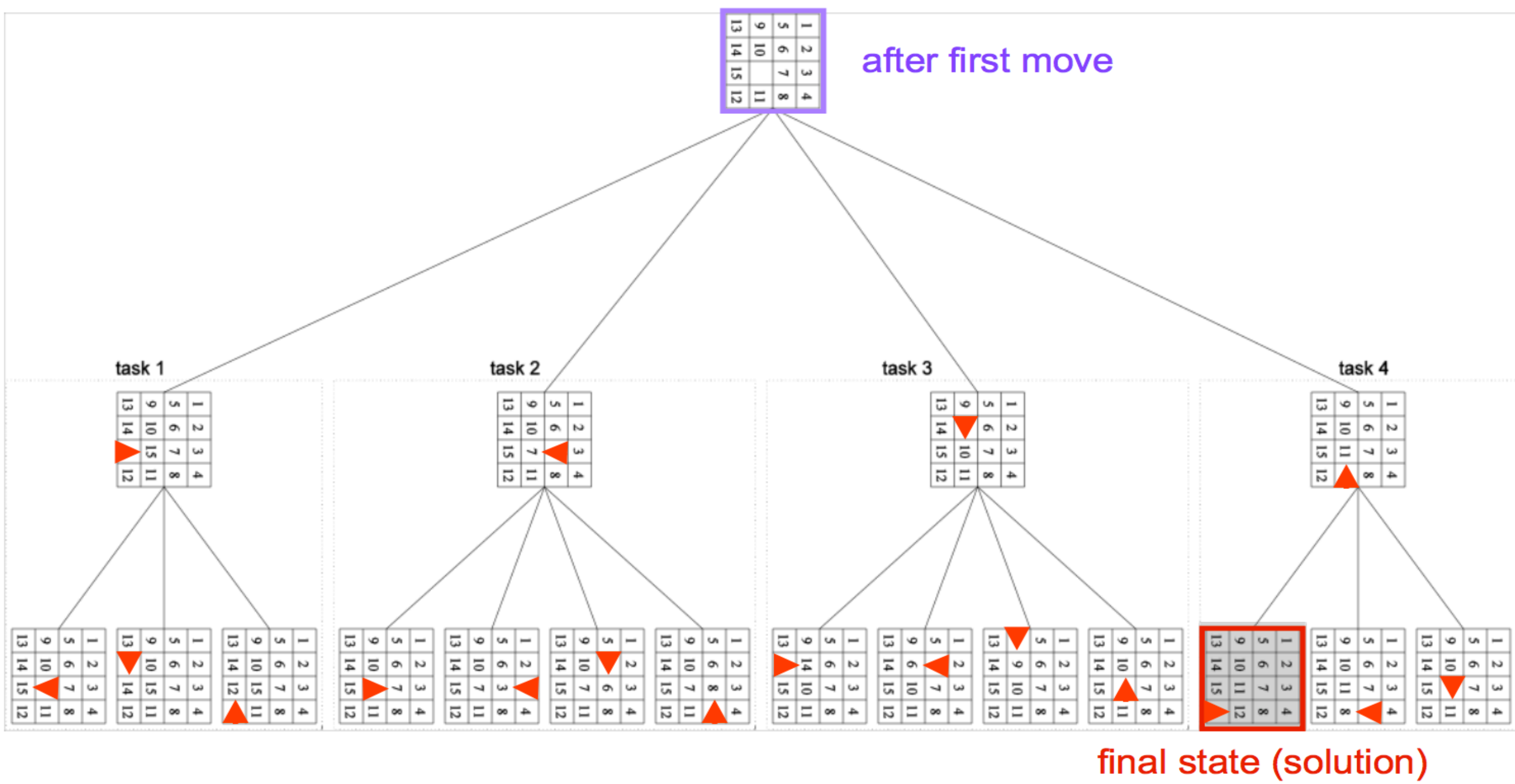
(a) (b) (c) (d)

- From an arbitrary state, must search for a solution

Exploratory Decomposition: Example

Solving a 15 puzzle

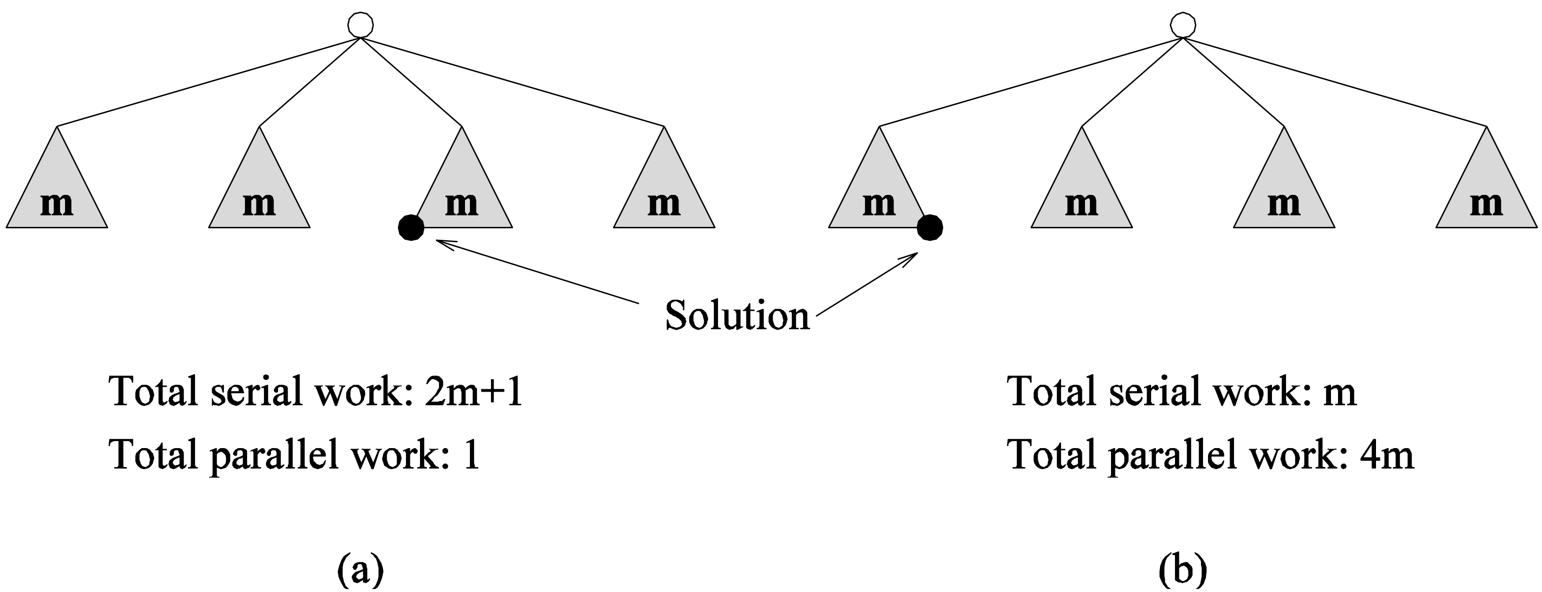
- Search
  - generate successor states of the current state
  - explore each as an independent task



Exploratory Decomposition Speedup

Solve a 15 puzzle

- The decomposition behaves according to the parallel formulation
  - May change the amount of work done



Execution terminate when a solution is found

Speculative Decomposition

- Dependencies between tasks are not known a-priori.
  - Impossible to identify independent tasks
- Two approaches
  - Conservative approaches, which identify independent tasks only when they are guaranteed to not have dependencies
    - May yield little concurrency
  - Optimistic approaches, which schedule tasks even when they may potentially be inter-dependent
    - Roll-back changes in case of an error

Speculative Decomposition: Example

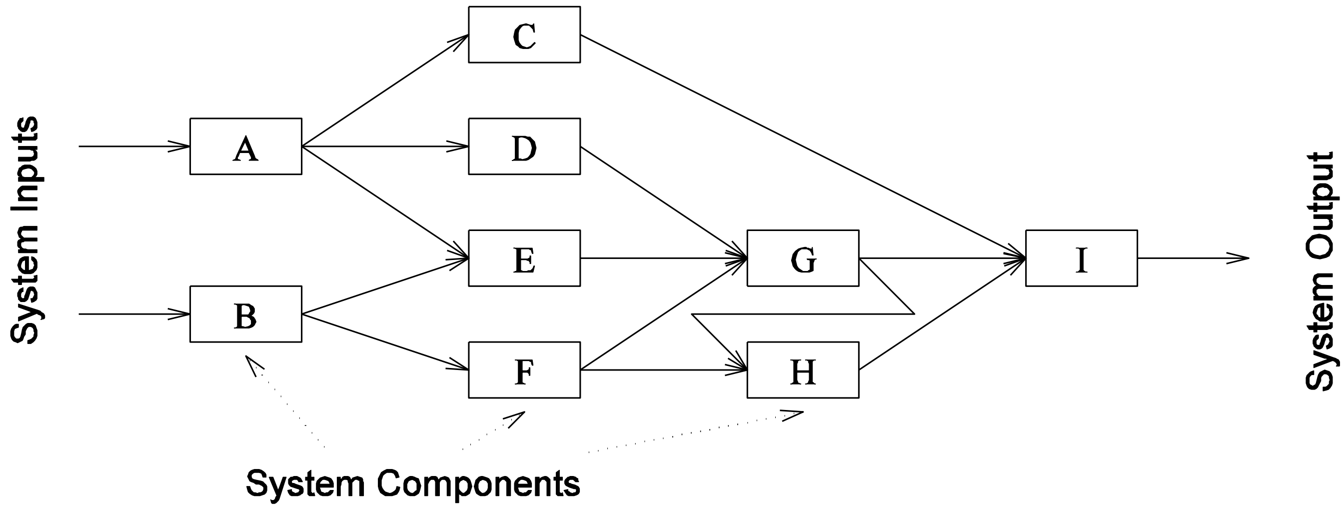
Discrete event simulation

- Centralized time-ordered event list
  - you get up →get ready→drive to work→work→eat lunch→ work some more→drive back→eat dinner→and sleep
- Simulation
  - extract next event in time order
  - process the event
  - if required, insert new events into the event list
- Optimistic event scheduling
  - assume outcomes of all prior events
  - speculatively process next event
  - if assumption is incorrect, roll back its effects and continue

Speculative Decomposition: Example

Simulation of a network of nodes

- Simulate network behavior for various input and node delays
  - The input are dynamically changing
    - Thus task dependency is unknown



- Speculate execution: tasks' input
  - Correct: parallelism
  - Incorrect: rollback and redo

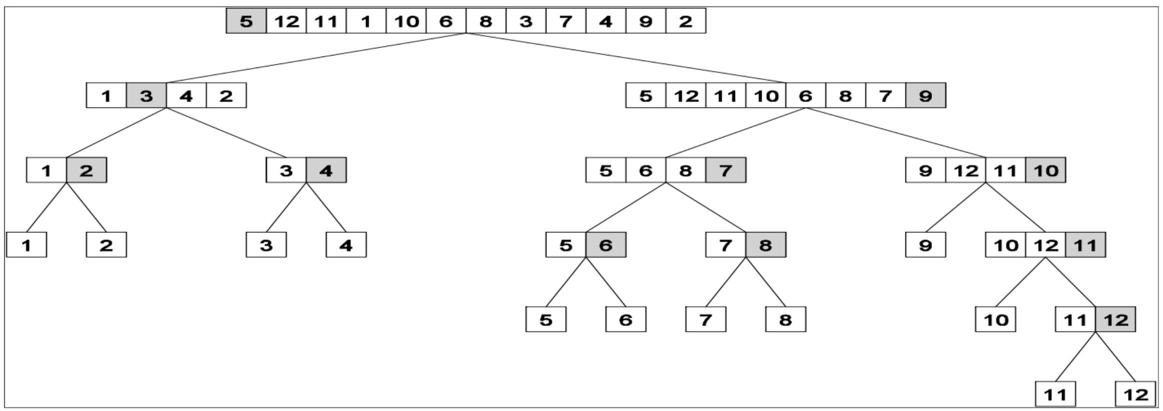
Speculative vs Exploratory

- Exploratory decomposition
  - The output of multiple tasks from a branch is unknown
  - Parallel program perform more, less or same amount of work as serial program
- Speculative
  - The input at a branch leading to multiple parallel tasks is unknown
  - Parallel program perform more or same amount of work as the serial algorithm

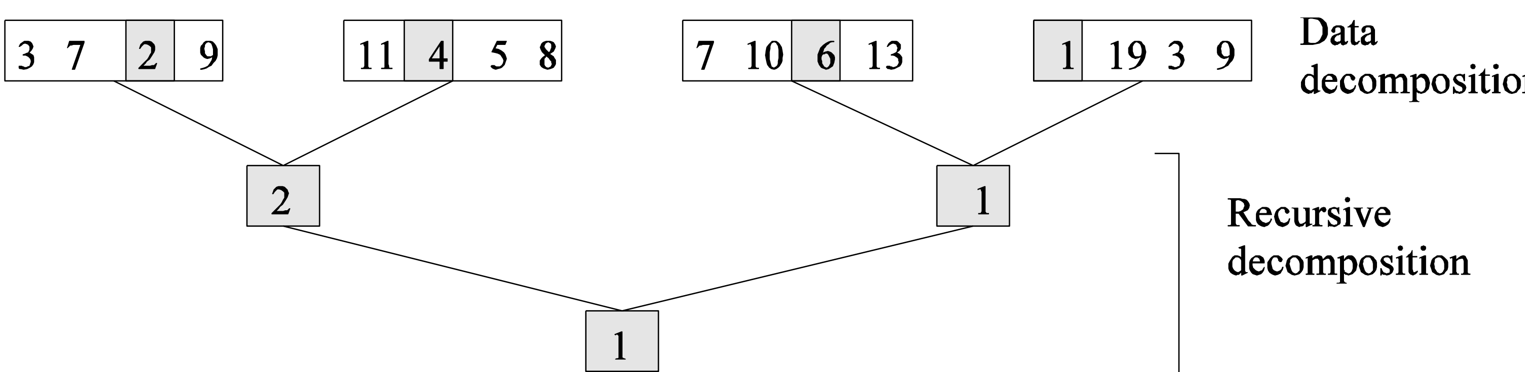
Hybrid Decompositions

Use multiple decomposition techniques together

- One decomposition may be not optimal for concurrency
  - Quicksort recursive decomposition limits concurrency (Why?)



- Combined recursive and data decomposition for MIN



Today's lecture

- Decomposition Techniques - continued
  - Exploratory Decomposition
  - Hybrid Decomposition

Mapping tasks to processes/cores/CPU/PEs

- ☛ Characteristics of Tasks and Interactions
  - Task Generation, Granularity, and Context
  - Characteristics of Task Interactions
- Mapping Techniques for Load Balancing
  - Static and Dynamic Mapping
- Methods for Minimizing Interaction Overheads
- Parallel Algorithm Design Models

Characteristics of Tasks

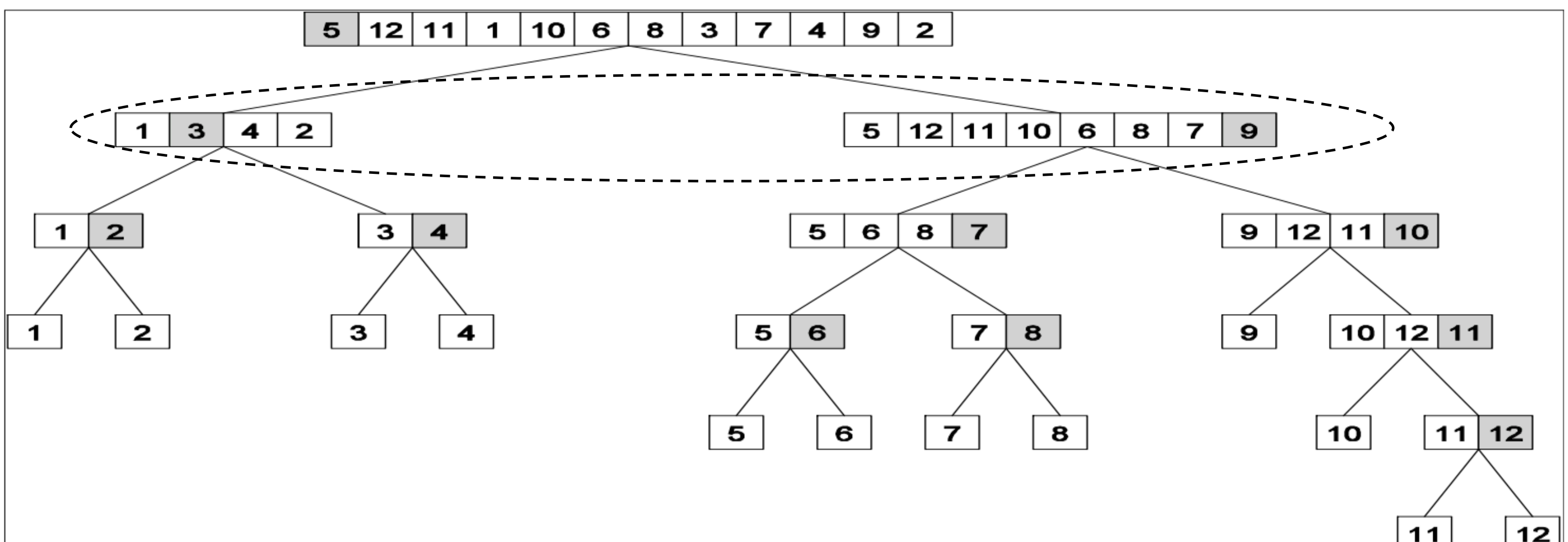
- Theory
  - Decomposition: to parallelize theoretically
    - Concurrency available in a problem
- Practice
  - Task creations, interactions and mapping to PEs.
    - Realizing concurrency practically
  - Characteristics of tasks and task interactions
    - Impact choice and performance of parallelism
- **Characteristics of tasks**
  - **Task generation strategies**
  - **Task sizes (the amount of work, e.g. FLOPs)**
  - **Size of data associated with tasks**

Task Generation

- Static task generation
  - Concurrent tasks and task graph known a-priori (before execution)
  - Typically using recursive or data decomposition
  - Examples
    - Matrix operations
    - Graph algorithms
    - Image processing applications
    - Other ***regularly*** structured problems
- Dynamic task generation
  - Computations formulate concurrent tasks and task graph on the fly
    - Not explicit a priori, though high-level rules or guidelines known
  - Typically by exploratory or speculative decompositions.
    - Also possible by recursive decomposition, e.g. quicksort
  - A classic example: game playing
    - 15 puzzle board

Task Sizes/Granularity

- The amount of work → amount of time to complete
  - E.g. FLOPs, #memory access
- Uniform:
  - Often by ***even*** data decomposition, i.e. regular
- Non-uniform
  - Quicksort, the choice of pivot



Size of Data Associated with Tasks

- May be small or large compared to the task sizes
  - How relevant to the input and/or output data sizes
  - Example:
    - **size(input) < size(computation), e.g., 15 puzzle**
    - **size(input) = size(computation) > size(output), e.g., min**
    - **size(input) = size(output) < size(computation), e.g., sort**
- Considering the efforts to reconstruct the same task context
  - small data: small efforts: task can easily migrate to another process
  - large data: large efforts: ties the task to a process
- Context reconstructing vs communicating
  - It depends

Characteristics of Task Interactions

- Aspects of interactions
  - What: shared data or synchronizations, and sizes of the media
  - When: the timing
  - Who: with which task(s), and overall topology/patterns
  - Do we know details of the above three before execution
  - How: involve one or both?
    - The implementation concern, implicit or explicit

Orthogonal classification

- Static vs. dynamic
- Regular vs. irregular
- Read-only vs. read-write
- One-sided vs. two-sided

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- Static interactions
  - Partners and timing (and else) are known a-priori
  - Relatively simpler to code into programs.
- Dynamic interactions
  - The timing or interacting tasks cannot be determined a-priori.
  - Harder to code, especially using explicit interaction.

Characteristics of Task Interactions

- Aspects of interactions
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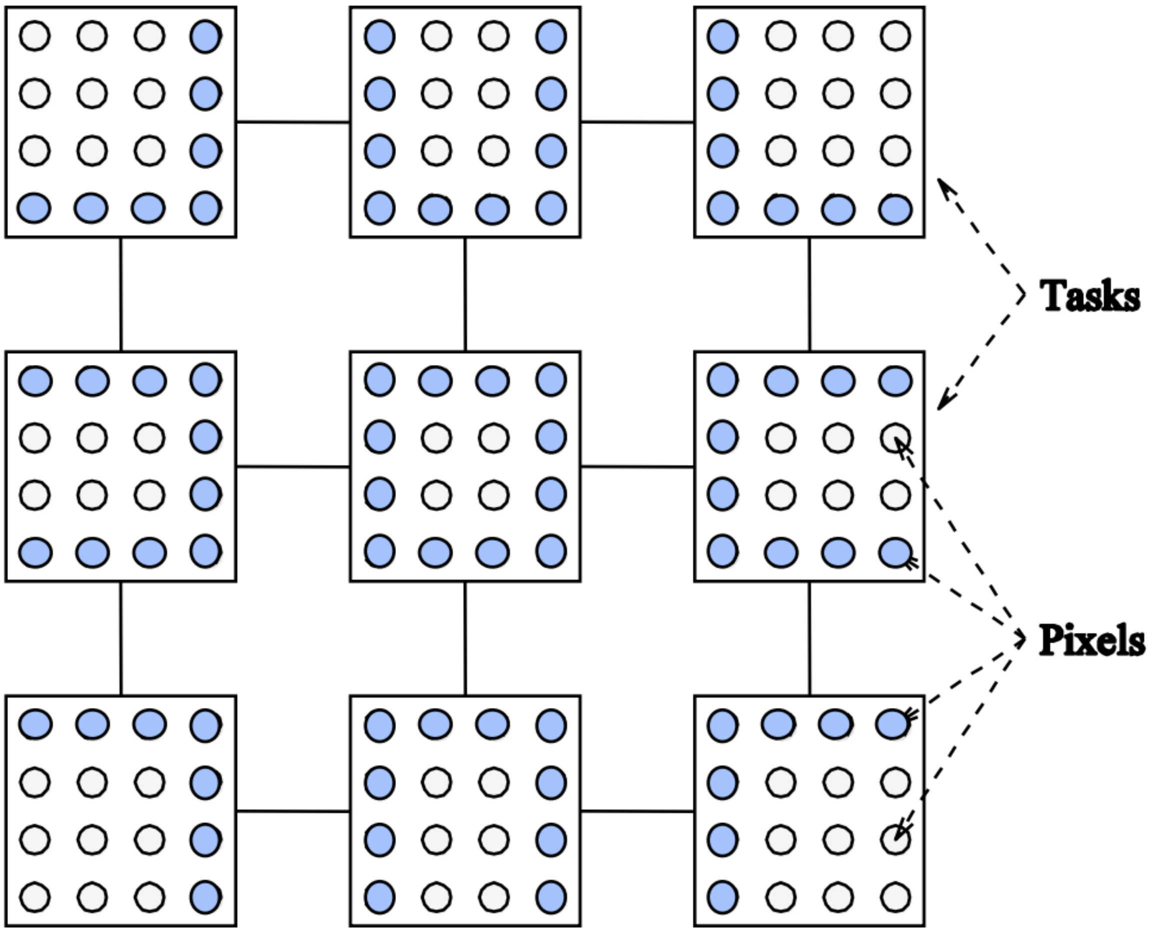
- Regular interactions
  - Definite pattern of the interactions
    - E.g. a mesh or ring
  - Can be exploited for efficient implementation.
- Irregular interactions
  - lack well-defined topologies
  - Modeled as a graph

Example of *Regular* Static Interaction

Image processing algorithms: dithering, edge detection

- Nearest neighbor interactions on a 2D mesh



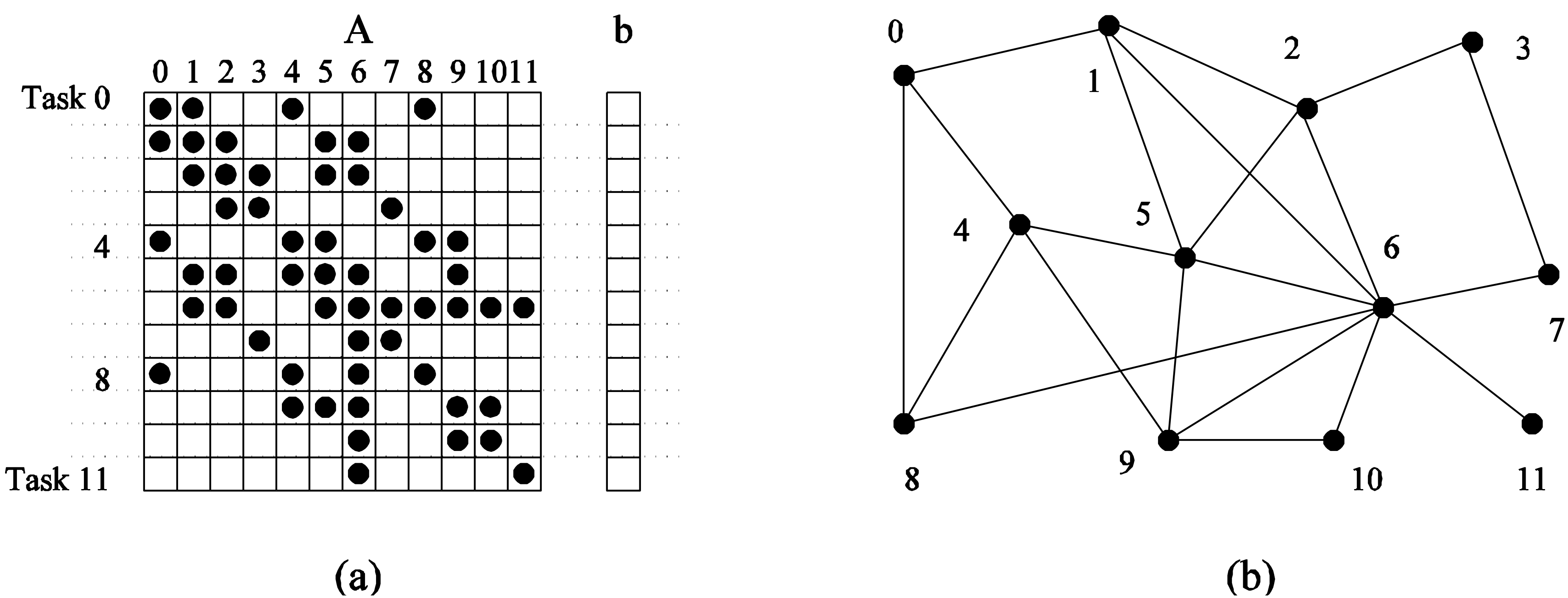
Sobel Edge Detection Stencils

$$\mathbf{G}_x = \begin{bmatrix} -1 & 0 & +1 \\ -2 & 0 & +2 \\ -1 & 0 & +1 \end{bmatrix}$$

$$\mathbf{G}_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ +1 & +2 & +1 \end{bmatrix}$$

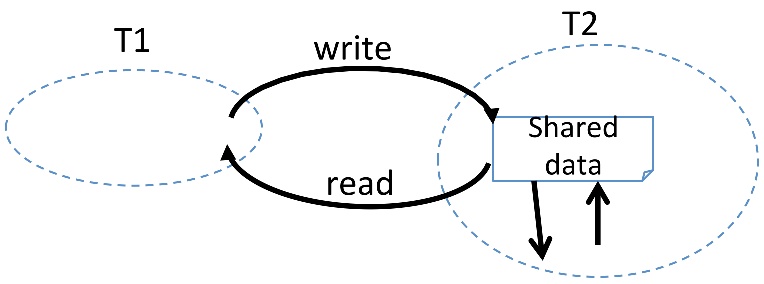
Example of *Irregular* Static Interaction

Sparse matrix vector multiplication



Characteristics of Task Interactions

- Aspects of interactions
  - What: shared data or synchronizations, and sizes of the media



- Read-only interactions
  - Tasks only read data items associated with other tasks
- Read-write interactions
  - Read, as well as modify data items associated with other tasks.
  - Harder to code
    - Require additional synchronization primitives
      - to avoid read-write and write-write ordering races

Characteristics of Task Interactions

- Aspects of interactions
  - What: shared data or synchronizations, and sizes of the media
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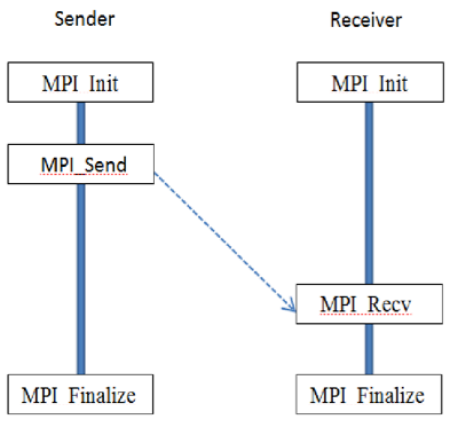
- One-sided
  - initiated & completed independently by 1 of 2 interacting tasks
    - GET and PUT
- Two-sided
  - both tasks coordinate in an interaction
    - SEND + RECV



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  - Exploratory Decomposition
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- Parallel Algorithm Design Models

Mapping Techniques

- Parallel algorithm design
  - Program decomposed
  - Characteristics of task and interactions identified

Assign large amount of concurrent tasks to equal or relatively small amount of processes for execution

- **Though often we do 1:1 mapping**

Mapping Techniques

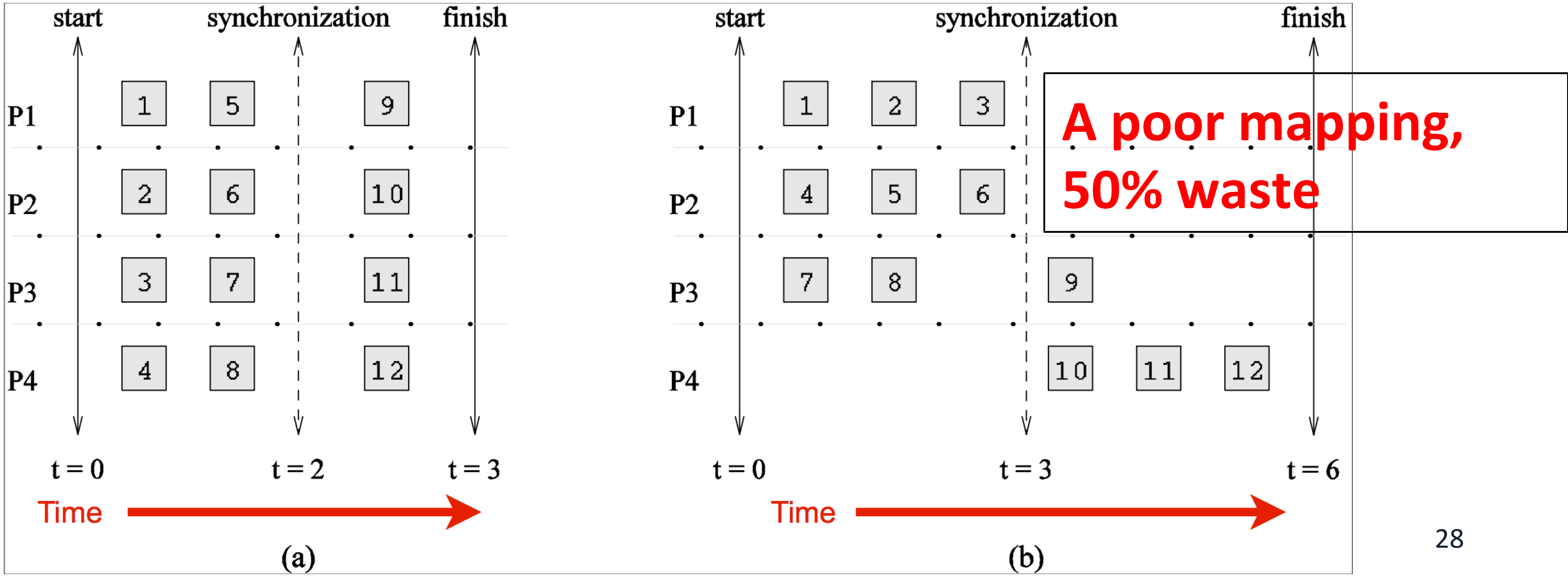
- Goal of mapping: minimize overheads
  - There is cost to do parallelism
    - **Interactions** and **idling(serialization)**

- Contradicting objectives: interactions vs idling
  - Idling (serialization) ⇧: insufficient parallelism
  - Interactions ⇧: excessive concurrency

  - E.g. Assigning all work to one processor trivially minimizes interaction at the expense of significant idling.

Mapping Techniques for Minimum Idling

- Execution: alternating stages of computation and interaction
- Mapping must simultaneously minimize idling and load balance
  - Idling means not doing useful work
  - Load balance: doing the same amount of work
- Merely balancing load does not minimize idling



Mapping Techniques for Minimum Idling

Static or dynamic mapping

- Static Mapping
  - Tasks are mapped to processes a-prior
  - Need a good estimate of task sizes
  - Optimal mapping may be NP complete

- Dynamic Mapping
  - Tasks are mapped to processes at runtime
  - Because:
    - Tasks are generated at runtime
    - Their sizes are not known.

- Other factors determining the choice of mapping techniques
  - the size of data associated with a task
  - the characteristics of inter-task interactions
  - even the programming models and target architectures

Schemes for Static Mapping

- Mappings based on data decomposition
  - Mostly 1-1 mapping

- Mappings based on task graph partitioning
- Hybrid mappings

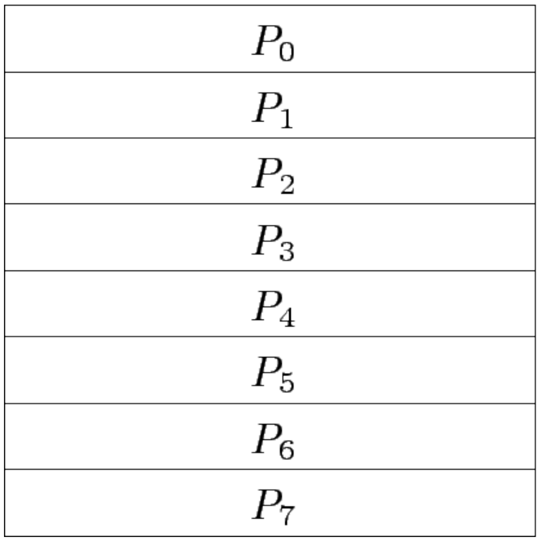
Mappings Based on Data Partitioning

- Partition the computation using a combination of
  - Data decomposition
  - The ``owner-computes'' rule

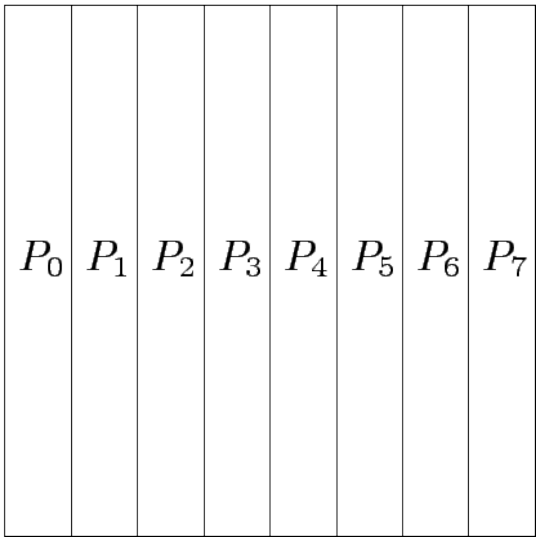
Example: 1-D *block distribution* of 2-D dense matrix

1-1 mapping of task/data and process

row-wise distribution

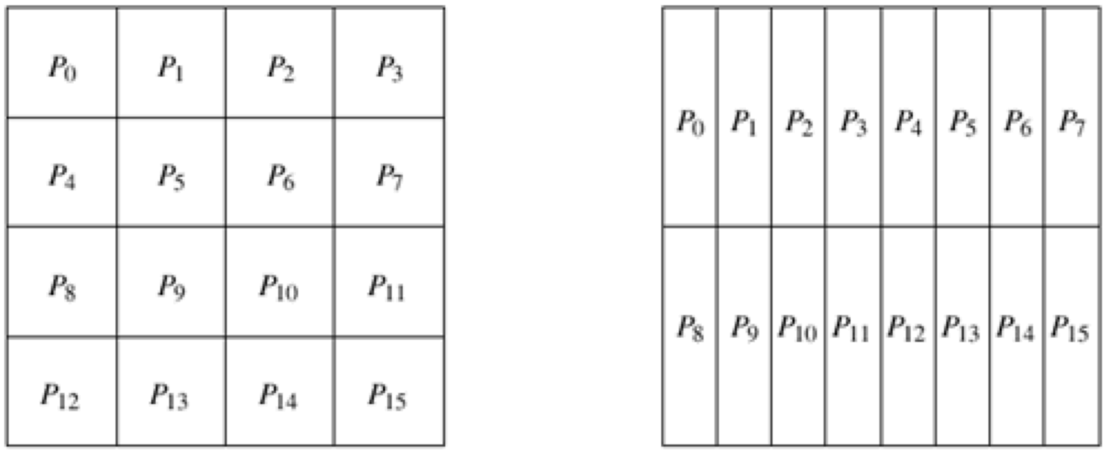


column-wise distribution

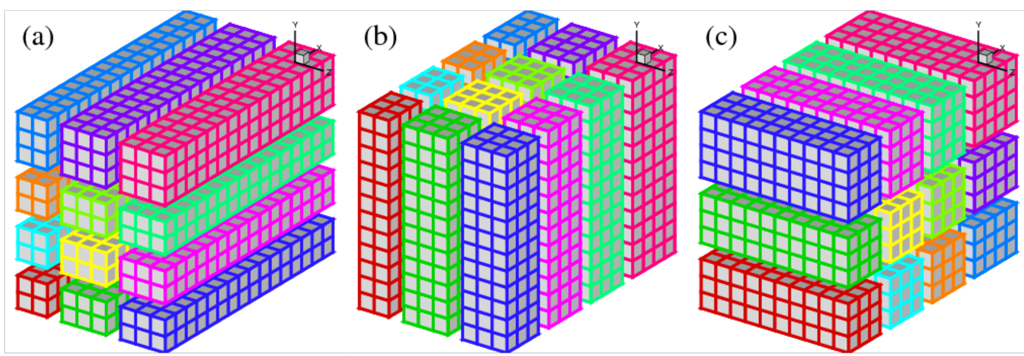


Block Array Distribution Schemes

Multi-dimensional Block distribution



(a) **(b)**



In general, higher dimension decomposition allows the use of larger # of processes.

Block Array Distribution Schemes: Examples

Multiplying two dense matrices: *A * B = C*

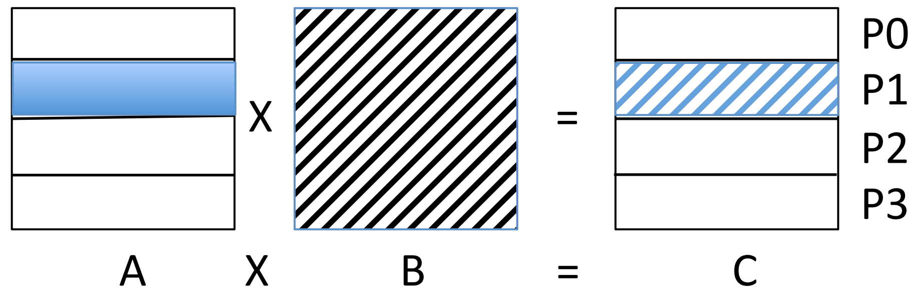
- Partition the output matrix ***C*** using a block decomposition
  - Load balance: Each task compute the same number of elements of ***C***
    - Note: each element of ***C*** corresponds to a single dot product
  - The choice of precise decomposition: 1-D (row/col) or 2-D
    - Determined by the associated communication overhead

$$\begin{bmatrix} A(11) & A(12) & A(13) \\ A(21) & A(22) & A(23) \\ A(31) & A(32) & A(33) \end{bmatrix} * \begin{bmatrix} B(11) & B(12) & B(13) \\ B(21) & B(22) & B(23) \\ B(31) & B(32) & B(33) \end{bmatrix} = \begin{bmatrix} C(11) & C(12) & C(13) \\ C(21) & C(22) & C(23) \\ C(31) & C(32) & C(33) \end{bmatrix}$$

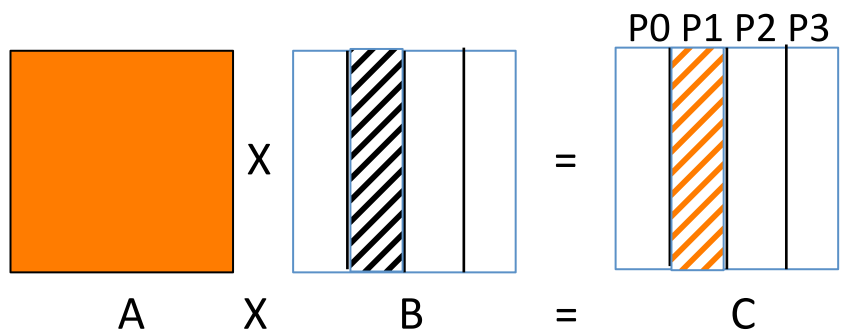
```
C(11) = A(11)*B(11) + A(12)*B(21) + A(13)*B(31)
C(21) = A(21)*B(11) + A(22)*B(21) + A(23)*B(31)
C(31) = A(31)*B(11) + A(32)*B(21) + A(33)*B(31)

C(12) = A(11)*B(12) + A(12)*B(22) + A(13)*B(32)
C(22) = A(21)*B(12) + A(22)*B(22) + A(23)*B(32)
C(32) = A(31)*B(12) + A(32)*B(22) + A(33)*B(32)

C(13) = A(11)*B(13) + A(12)*B(23) + A(13)*B(33)
C(23) = A(21)*B(13) + A(22)*B(23) + A(23)*B(33)
C(33) = A(31)*B(13) + A(32)*B(23) + A(33)*B(33)
```

Block Distribution and Data Sharing for Dense Matrix Multiplication

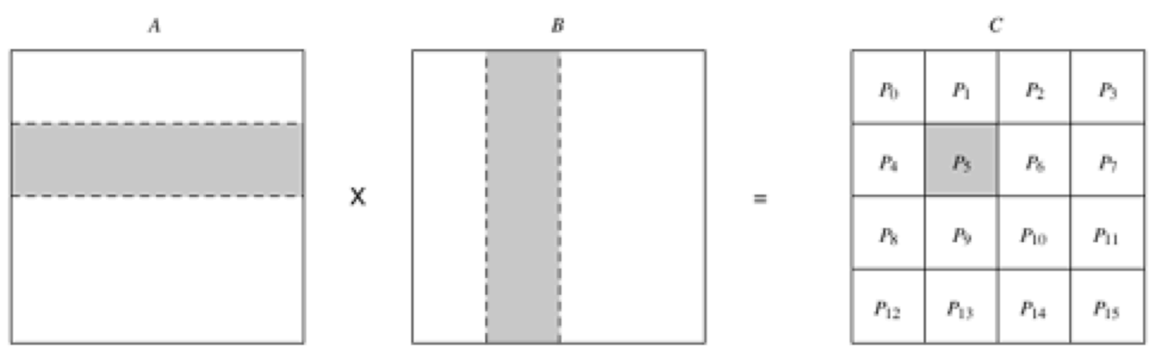
- Row-based 1-D



- Column-based 1-D

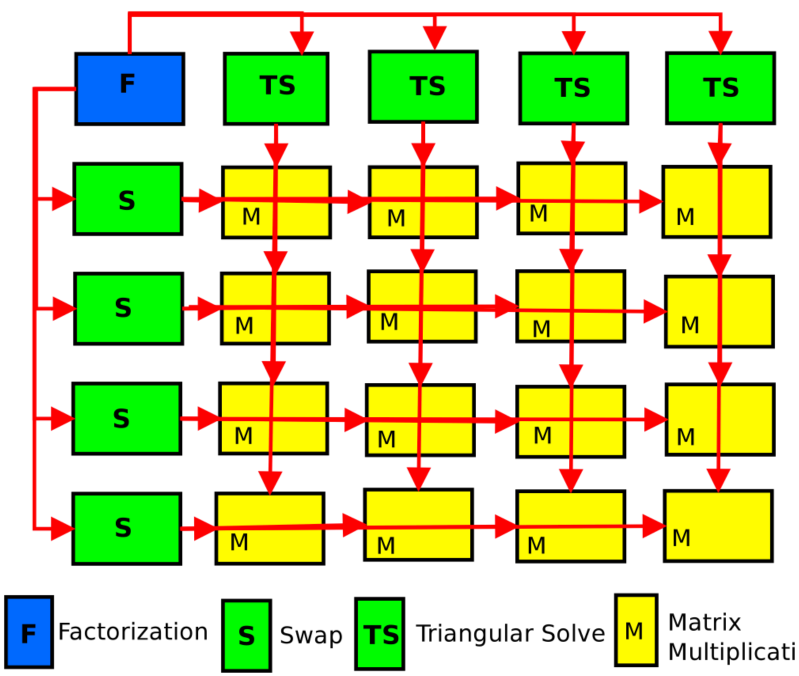


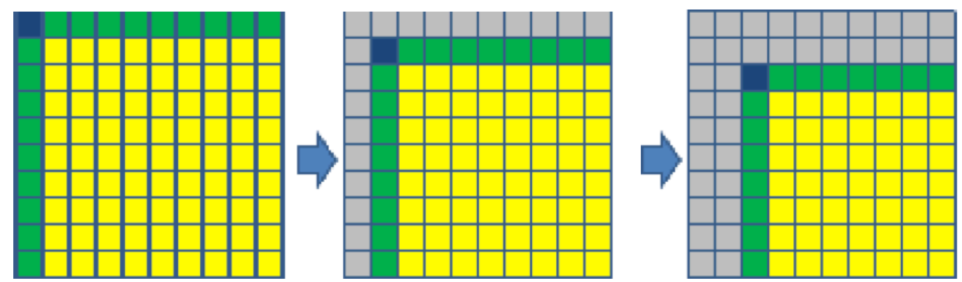
- Row/Col-based 2-D



Cyclic and Block Cyclic Distributions

- Consider a block distribution for LU decomposition (Gaussian Elimination)
  - **The amount of computation per data item varies**
  - **Block decomposition would lead to significant load imbalance**





```
procedure COL_LU (A)
begin
   for k := 1 to n do
       for j := k to n do
           A[j, k]:= A[j, k]/A[k, k];
       endfor;
       for j := k + 1 to n do
           for i := k + 1 to n do
               A[i, j] := A[i, j] - A[i, k] x A[k, j];
           endfor;
       endfor;
  /*
After this iteration, column A[k + 1 : n, k] is logically the kth
column of L and row A[k, k : n] is logically the kth row of U.
  */
```

LU Factorization of a Dense Matrix

A decomposition of LU factorization into 14 tasks

$$\begin{pmatrix} A_{1,1} & A_{1,2} & A_{1,3} \\ A_{2,1} & A_{2,2} & A_{2,3} \\ A_{3,1} & A_{3,2} & A_{3,3} \end{pmatrix} \rightarrow \begin{pmatrix} L_{1,1} & 0 & 0 \\ L_{2,1} & L_{2,2} & 0 \\ L_{3,1} & L_{3,2} & L_{3,3} \end{pmatrix} \cdot \begin{pmatrix} U_{1,1} & U_{1,2} & U_{1,3} \\ 0 & U_{2,2} & U_{2,3} \\ 0 & 0 & U_{3,3} \end{pmatrix}$$

1: $A_{1,1} \rightarrow L_{1,1}U_{1,1}$

2: $L_{2,1} = A_{2,1}U_{1,1}^{-1}$

3: $L_{3,1} = A_{3,1}U_{1,1}^{-1}$

4: $U_{1,2} = L_{1,1}^{-1}A_{1,2}$

5: $U_{1,3} = L_{1,1}^{-1}A_{1,3}$

6: $A_{2,2} = A_{2,2} - L_{2,1}U_{1,2}$

7: $A_{3,2} = A_{3,2} - L_{3,1}U_{1,2}$

8: $A_{2,3} = A_{2,3} - L_{2,1}U_{1,3}$

9: $A_{3,3} = A_{3,3} - L_{3,1}U_{1,3}$

10: $A_{2,2} \rightarrow L_{2,2}U_{2,2}$

11: $L_{3,2} = A_{3,2}U_{2,2}^{-1}$

12: $U_{2,3} = L_{2,2}^{-1}A_{2,3}$

13: $A_{3,3} = A_{3,3} - L_{3,2}U_{2,3}$

14: $A_{3,3} \rightarrow L_{3,3}U_{3,3}$

Block Distribution for LU

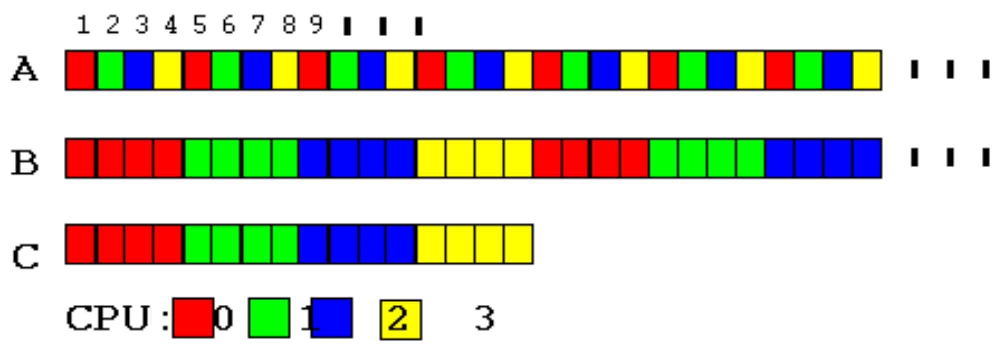
Notice the significant load imbalance

P_0 T_1	P_3 T_4	P_6 T_5
P_1 T_2	P_4 T_6 T_{10}	P_7 T_8 T_{12}
P_2 T_3	P_5 T_7 T_{11}	P_8 T_9 T_{13} T_{14}

Block Cyclic Distributions

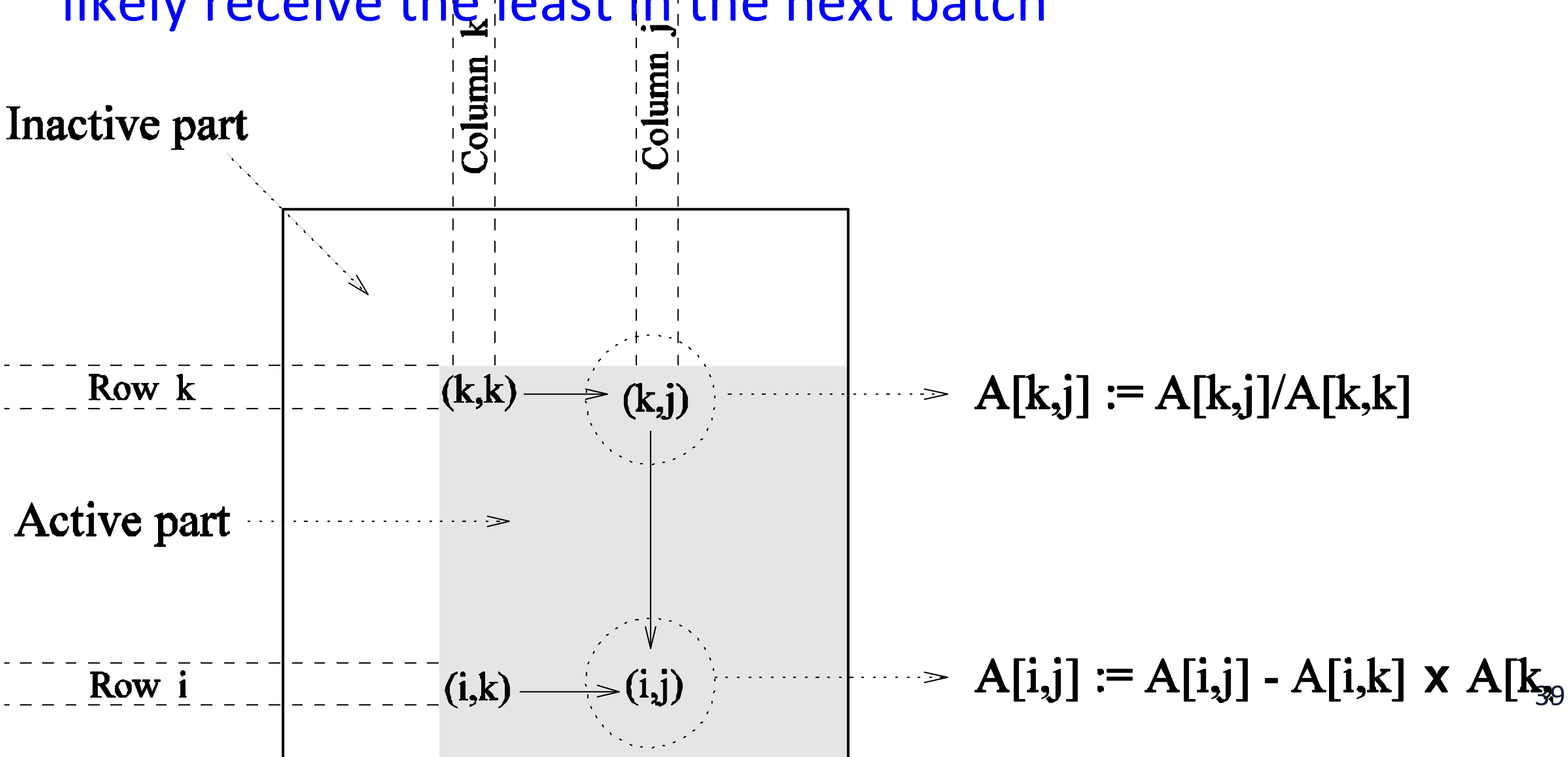
- Variation of the block distribution scheme
  - Partition an array into many more blocks (i.e. tasks) than the number of available processes.
  - Blocks are assigned to processes in a round-robin manner so that each process gets several non-adjacent blocks.
  - N-1 mapping of tasks to processes
- Used to alleviate the load-imbalance and idling problems.

```
      REAL, DIMENSION(N) :: A, B
      REAK, DIMENSION(12) :: C
!HPF$ DISTRIBUTE A(CYCLIC)
!HPF$ DISTRIBUTE B(CYCLIC(4))
!HPF$ DISTRIBUTE C(BLOCK)
```



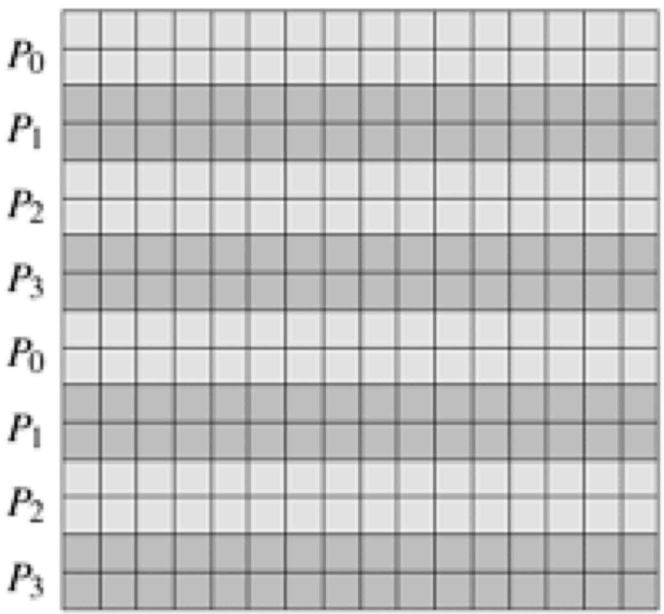
Block-Cyclic Distribution for Gaussian Elimination

- Active submatrix shrinks as elimination progresses
- Assigning blocks in a block-cyclic fashion
  - Each PEs receives blocks from different parts of the matrix
  - In one batch of mapping, the PE doing the most will most likely receive the least in the next batch

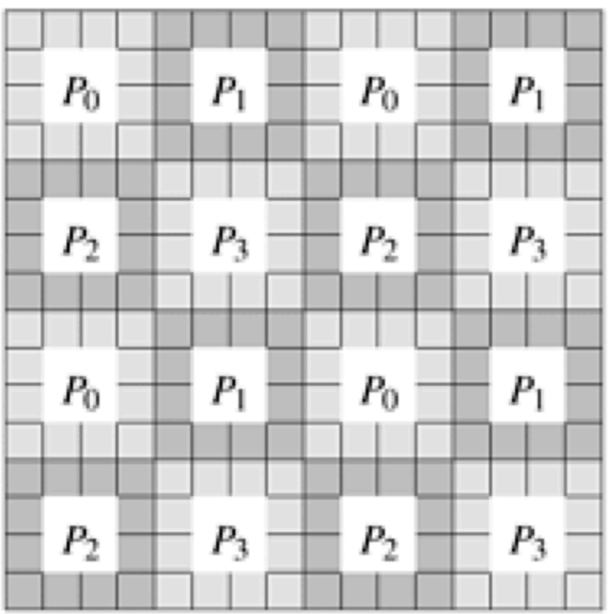


Block-Cyclic Distribution

- A cyclic distribution: a special case with block size = 1
- A block distribution: a special case with block size = ***n/p***
  - ***n*** is the dimension of the matrix and ***p*** is the #of processes.



(a)

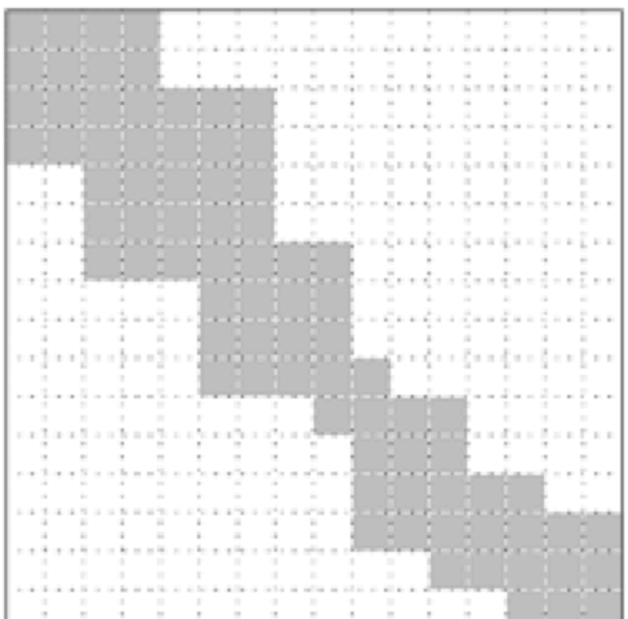


(b)

Block Partitioning and Random Mapping

Sparse matrix computations

- Load imbalance using block-cyclic partitioning/mapping
  - more non-zero blocks to diagonal processes *P*0, *P*5, *P*10, and *P*15 than others
  - P12 gets nothing



(a)

P_0	P_1	P_2	P_3	P_0	P_1	P_2	P_3
P_4	P_5	P_6	P_7	P_4	P_5	P_6	P_7
P_8	P_9	P_{10}	P_{11}	P_8	P_9	P_{10}	P_{11}
P_{12}	P_{13}	P_{14}	P_{15}	P_{12}	P_{13}	P_{14}	P_{15}
P_0	P_1	P_2	P_3	P_0	P_1	P_2	P_3
P_4	P_5	P_6	P_7	P_4	P_5	P_6	P_7
P_8	P_9	P_{10}	P_{11}	P_8	P_9	P_{10}	P_{11}
P_{12}	P_{13}	P_{14}	P_{15}	P_{12}	P_{13}	P_{14}	P_{15}

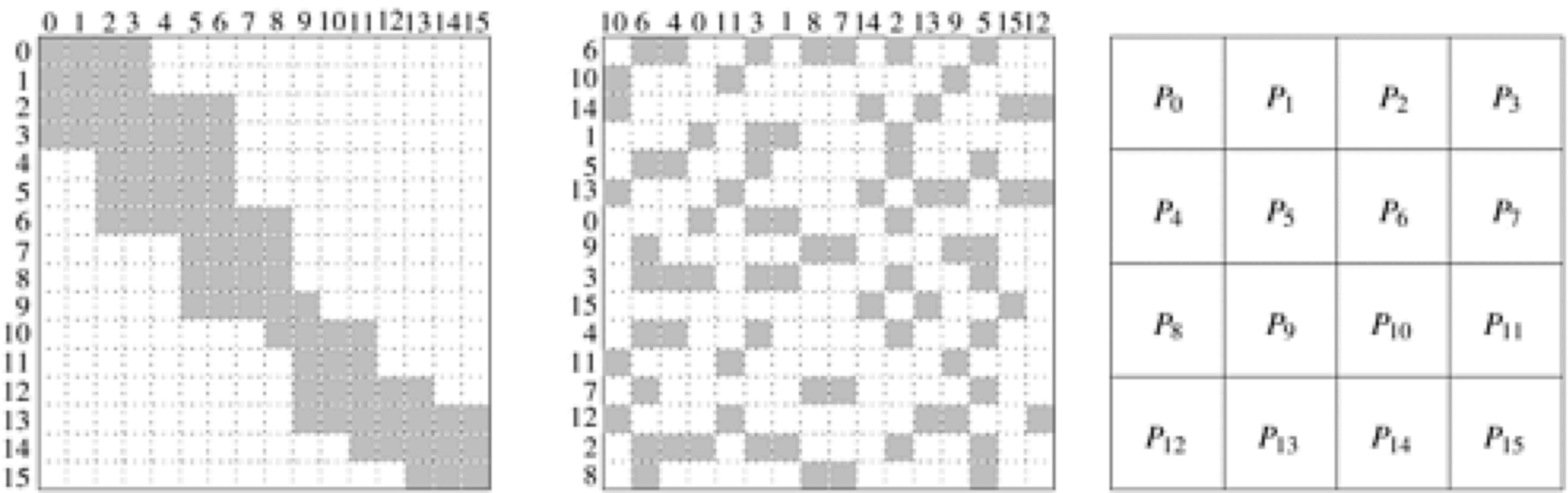
(b)

Block Partitioning and Random Mapping

V = [0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]

random(V) = [8, 2, 6, 0, 3, 7, 11, 1, 9, 5, 4, 10]

mapping = $\underbrace{8\ \ 2\ \ 6}_{P_0}\ \underbrace{0\ \ 3\ \ 7}_{P_1}\ \underbrace{11\ \ 1\ \ 9}_{P_2}\ \underbrace{5\ \ 4\ \ 10}_{P_3}$



(a) (b) (c)

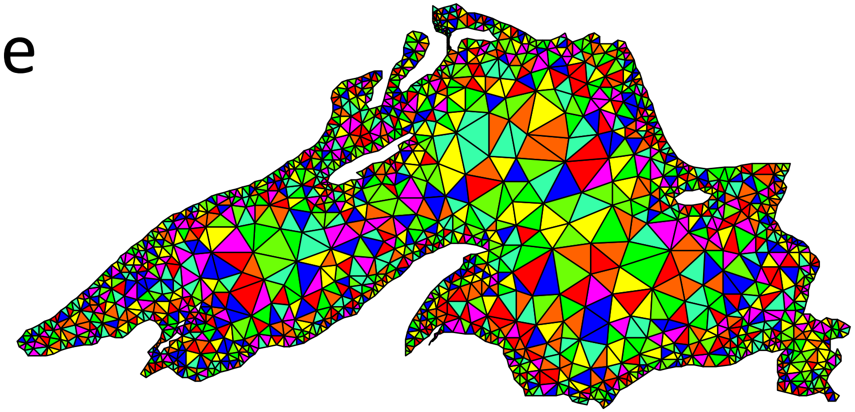
Graph Partitioning Based Data Decomposition

- Array-based partitioning and static mapping
  - Regular domain, i.e. rectangular, mostly dense matrix
  - Structured and regular interaction patterns
  - Quite effective in balancing the computations and minimizing the interactions

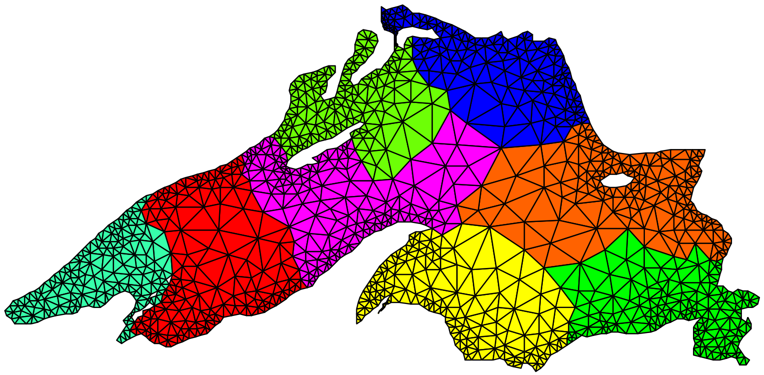
- Irregular domain
  - Spars matrix-related
  - Numerical simulations of physical phenomena
    - Car, water/blood flow, geographic
- Partition the irregular domain so as to
  - Assign equal number of nodes to each process
  - Minimizing edge count of the partition.

Partitioning the Graph of Lake Superior

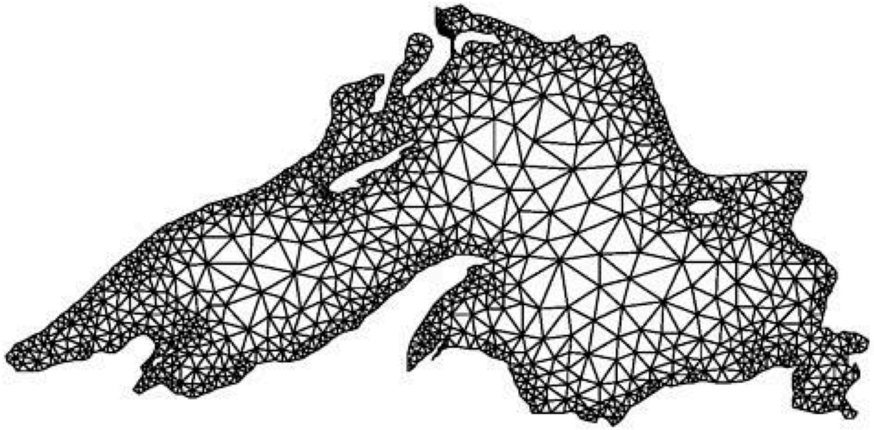
- Each mesh point has the same amount of computation
  - Easy for load balancing
- Minimize edges
- Optimal partition is an NP-complete
  - Use heuristics



Random Partitioning



Partitioning for minimum edge-cut.



Mappings Based on Task Paritioning

- Schemes for Static Mapping
  - Mappings based on data partitioning
    - Mostly 1-1 mapping
  - Mappings based on task graph partitioning
  - Hybrid mappings

- Data partitioning
  - **Data** decomposition and then 1-1 mapping of tasks to PEs

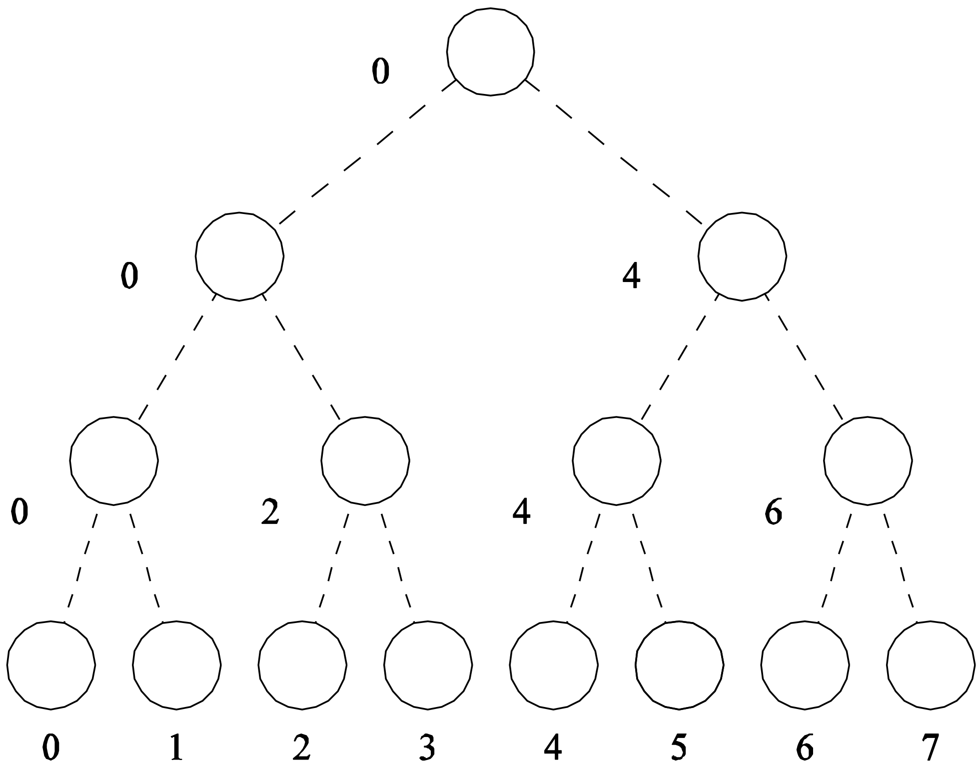
Partitioning a given task-dependency graph across processes

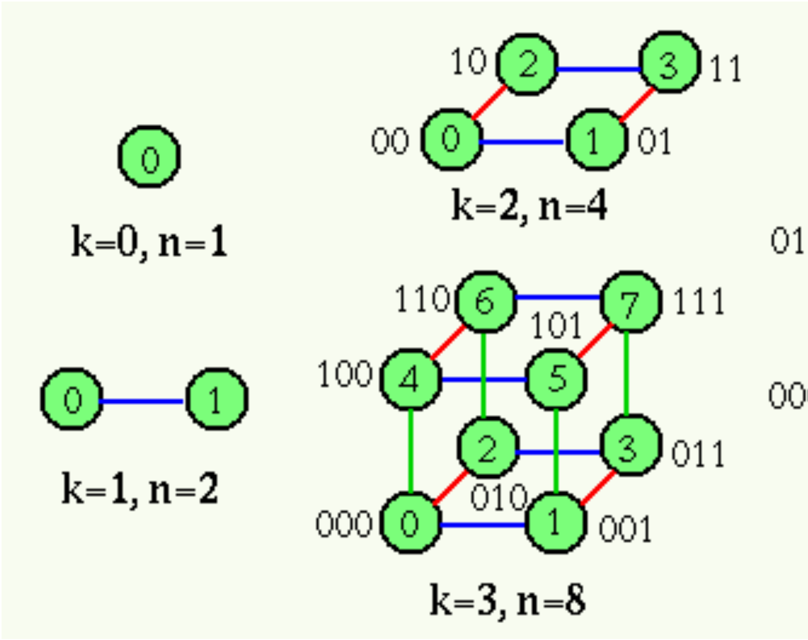
- An optimal mapping for a general task-dependency graph
  - NP-complete problem.
- Excellent heuristics exist for structured graphs.

Mapping a Binary Tree Dependency Graph

Mapping dependency graph of quicksort to processes in a hypercube

- Hypercube: n-dimensional analogue of a square and a cube
  - node numbers that differ in 1 bit are adjacent

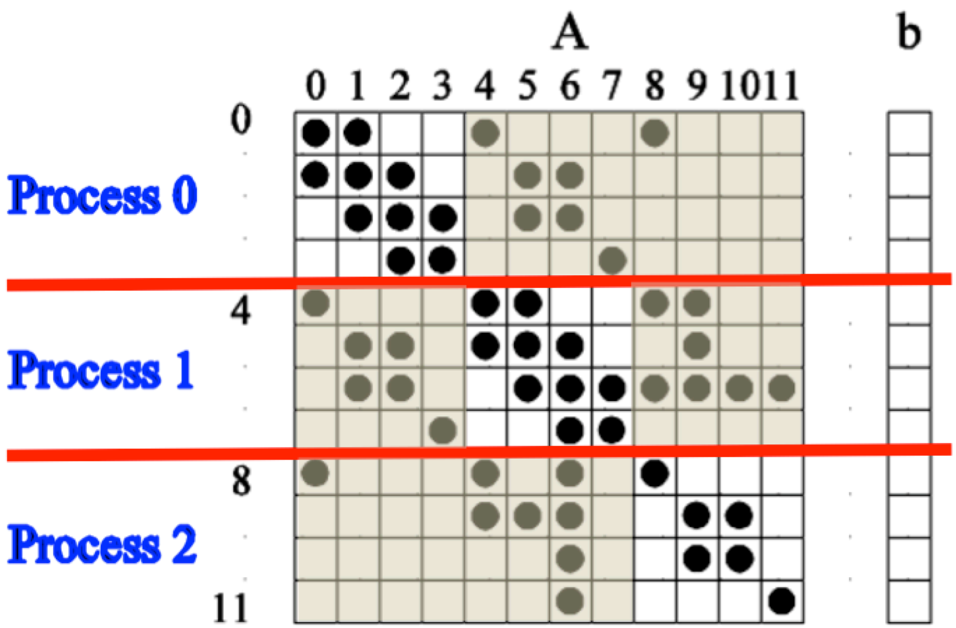




Mapping a Sparse Graph

Sparse matrix vector multiplication

Using data partitioning



17 items to communicate

C0 = (4,5,6,7,8)

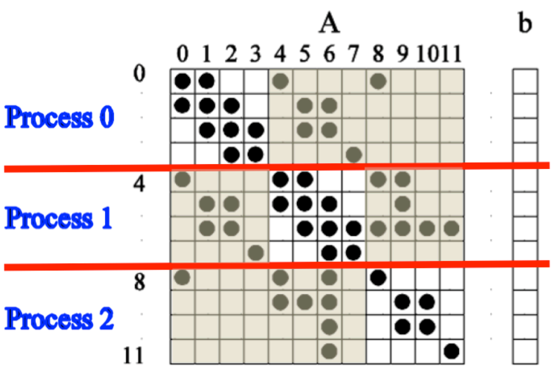
C1 = (0,1,2,3,8,9,10,11)

C2 = (0,4,5,6)

Mapping a Sparse Graph

Sparse matrix vector multiplication

Using task graph partitioning



17 items to communicate

C0 = (4,5,6,7,8)

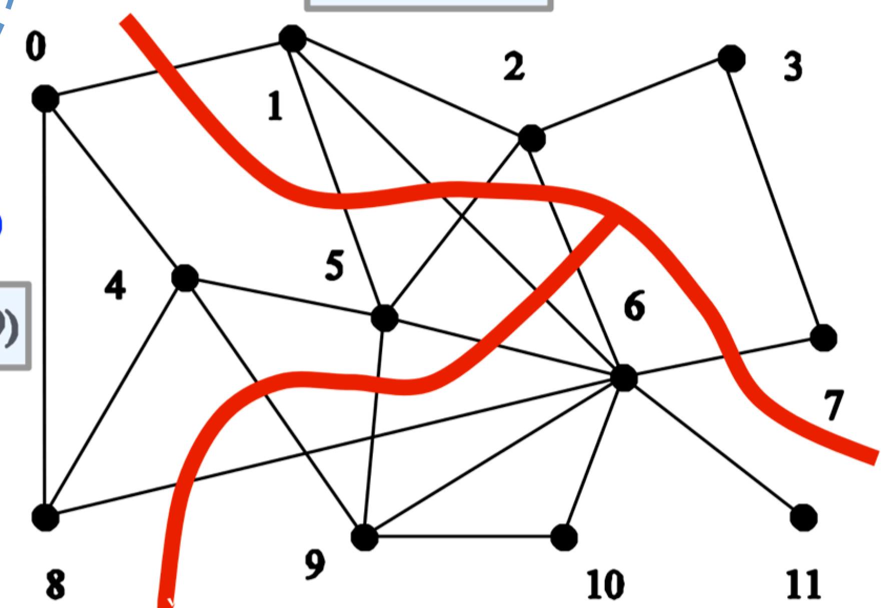
C1 = (0,1,2,3,8,9,10,11)

C2 = (0,4,5,6)

13 items to communicate

C1 = (0,5,6)

Process 1



Process 0

C0 = (1,2,6,9)

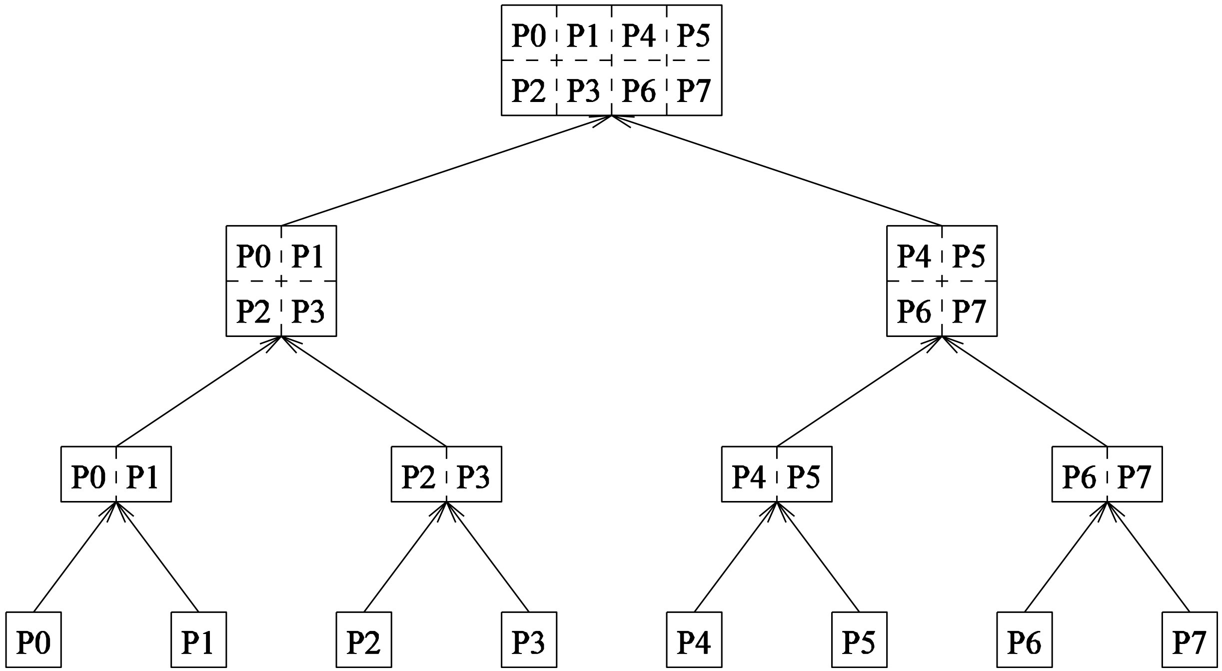
Process 2

C2 = (1,2,4,5,7,8)

Process 0	0,4,5,8
Process 1	1,2,3,7
Process 2	6,9,10,11

Hierarchical/Hybrid Mappings

- A single mapping is inadequate.
  - E.g. task graph mapping of the binary tree (quicksort) cannot use a large number of processors.
- Hierarchical mapping
  - Task graph mapping at the top level
  - Data partitioning within each level.



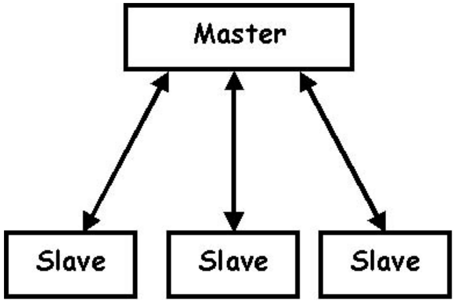
Today's lecture

- Decomposition Techniques - continued
  - Exploratory Decomposition
  - Hybrid Decomposition
- Characteristics of Tasks and Interactions
  - Task Generation, Granularity, and Context
  - Characteristics of Task Interactions
- Mapping Techniques for Load Balancing
  - Static
  - Dynamic Mapping
- Methods for Minimizing Interaction Overheads
- Parallel Algorithm Design Models

Schemes for Dynamic Mapping

- Also referred to as dynamic load balancing
  - Load balancing is the primary motivation for dynamic mapping.
- Dynamic mapping schemes can be
  - Centralized
  - Distributed

Centralized Dynamic Mapping



- Processes are designated as **masters** or **slaves**
  - Workers (slave is politically incorrect)
- General strategies
  - Master has pool of tasks and as central dispatcher
  - When one runs out of work, it requests from master for more work.
- Challenge
  - When process # increases, master may become the bottleneck.
- Approach
  - Chunk scheduling: a process picks up multiple tasks at once
  - Chunk size:
    - Large chunk sizes may lead to significant load imbalances as well
    - Schemes to gradually decrease chunk size as the computation progresses.

Distributed Dynamic Mapping

- All processes are created equal
  - Each can send or receive work from others
    - Alleviates the bottleneck in centralized schemes.
- Four critical design questions:
  - how are sending and receiving processes paired together
  - who initiates work transfer
  - how much work is transferred
  - when is a transfer triggered?
- Answers are generally application specific.

- Workstealing

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Minimizing Interaction Overheads

Rules of thumb

- Maximize data locality
  - Where possible, reuse intermediate data
  - Restructure computation so that data can be reused in smaller time windows.
- Minimize volume of data exchange
  - partition interaction graph to minimize edge crossings
- Minimize frequency of interactions
  - Merge multiple interactions to one, e.g. aggregate small msgs.
- Minimize contention and hot-spots
  - Use decentralized techniques
  - Replicate data where necessary

Minimizing Interaction Overheads (continued)

Techniques

- Overlapping computations with interactions
  - Use non-blocking communications
  - Multithreading
  - Prefetching to hide latencies.
- Replicating data or computations to reduce communication
- Using group communications instead of point-to-point primitives.
- Overlap interactions with other interactions.

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Parallel Algorithm Models

- Ways of structuring parallel algorithm
  - Decomposition techniques
  - Mapping technique
  - Strategy to minimize interactions.

- Data Parallel Model
  - Each task performs similar operations on different data
  - Tasks are statically (or semi-statically) mapped to processes
- Task Graph Model
  - Use task dependency graph to guide the model for better locality or low interaction costs.

Parallel Algorithm Models (continued)

- Master-Slave Model
  - Master (one or more) generate work
  - Dispatch work to workers.
  - Dispatching may be static or dynamic.
- Pipeline / Producer-Comsumer Model
  - Stream of data is passed through a succession of processes, each of which perform some task on it
  - Multiple stream concurrently
- Hybrid Models
  - Applying multiple models hierarchically
  - Applying multiple models sequentially to different phases of a parallel algorithm.

References

- Adapted from slides "Principles of Parallel Algorithm Design" by Ananth Grama
- Based on Chapter 3 of "Introduction to Parallel Computing" by Ananth Grama, Anshul Gupta, George Karypis, and Vipin Kumar. Addison Wesley, 2003