# Lecture 7X: Practices with Principles of Parallel Algorithm Design

**Concurrent and Multicore Programming**
**CSE 436/536**


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# Short Review and Today's Class

- Parallel Algorithms
  1. **Tasks and Decomposition**
  2. **Processes and Mapping**
  3. **Minimizing Interaction Overheads**

- Practice on **data decomposition** with working examples
  - BLAS and linear algebra
  - AXPY, Matrix vector multiplication, matrix matrix multiplication
- Practice on running examples, and collect and report performance results
  - See examples

# Review of Last Class Contents

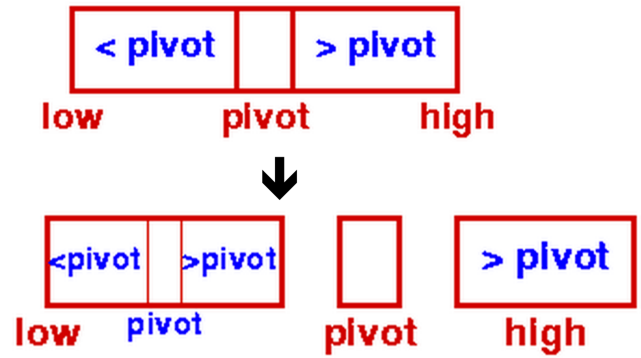
## Decomposing a large problem into multiple smaller one (tasks)

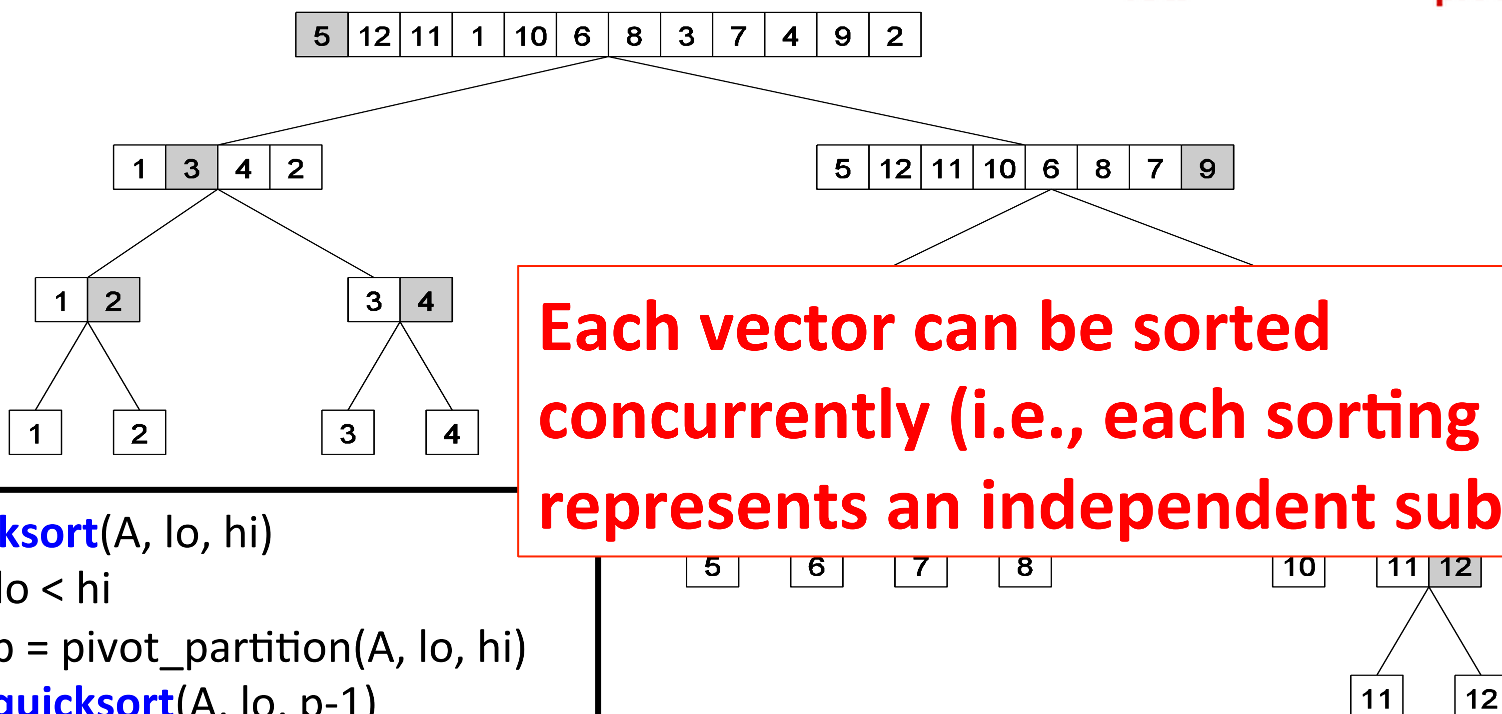
- Recursive Decomposition
- Data Decomposition

# Recursive Decomposition: Quicksort

At each level and for each vector

1. Select a pivot
2. Partition set around pivot
3. Recursively sort each subvector




**Each vector can be sorted concurrently (i.e., each sorting represents an independent subtask).**

```
quicksort(A, lo, hi)
  if lo < hi
    p = pivot_partition(A, lo, hi)
    quicksort(A, lo, p-1)
    quicksort(A, p+1, hi)
```

# Recursive Decomposition: Min

## Finding the minimum in a vector using divide-and-conquer

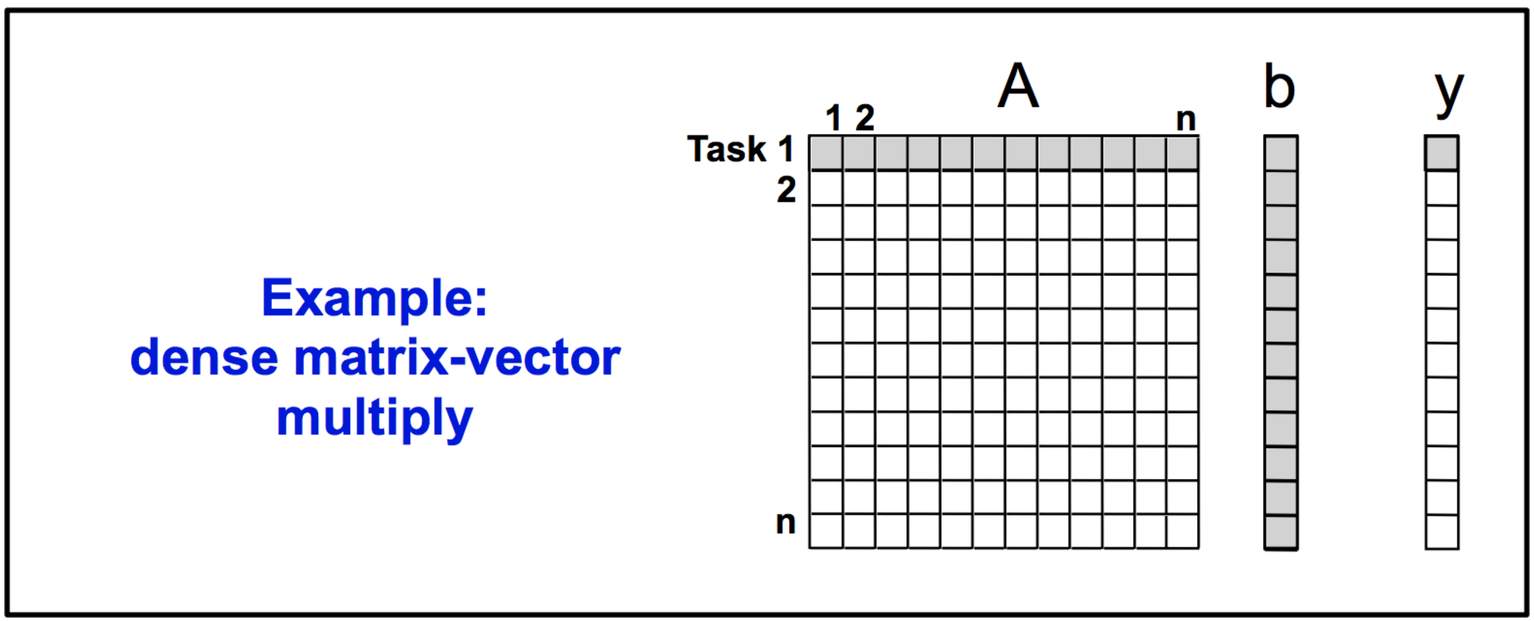
```
procedure SERIAL_MIN (A, n)
    min = A[0];
    for i := 1 to n − 1 do
        if (A[i] < min) min := A[i];
    return min;
```

```
procedure RECURSIVE_MIN (A, n)
    if ( n = 1 ) then min := A [0]  ;
    else
        lmin := RECURSIVE_MIN (A, n/2 );
        rmin := RECURSIVE_MIN (&(A[n/2]), n - n/2);
        if (lmin  < rmin) then min := lmin;
        else min := rmin;
    return min;
```

**Applicable to other associative operations, e.g. sum, AND …**

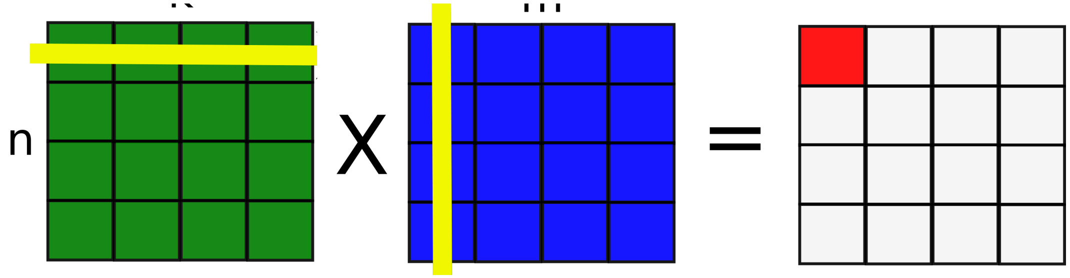
# Output Data Decomposition

- Each element of the output can be computed independently of others
  - simply as a function of the input.
- A natural problem decomposition



# Output Data Decomposition: Matrix Multiplication

**multiplying two *n* x *n* matrices *A* and *B* to yield matrix *C***




The output matrix ***C*** can be partitioned into four tasks:

$$\begin{pmatrix} A_{1,1} & A_{1,2} \\ A_{2,1} & A_{2,2} \end{pmatrix} \cdot \begin{pmatrix} B_{1,1} & B_{1,2} \\ B_{2,1} & B_{2,2} \end{pmatrix} \rightarrow \begin{pmatrix} C_{1,1} & C_{1,2} \\ C_{2,1} & C_{2,2} \end{pmatrix}$$

Task 1: $C_{1,1} = A_{1,1}B_{1,1} + A_{1,2}B_{2,1}$

Task 2: $C_{1,2} = A_{1,1}B_{1,2} + A_{1,2}B_{2,2}$

Task 3: $C_{2,1} = A_{2,1}B_{1,1} + A_{2,2}B_{2,1}$

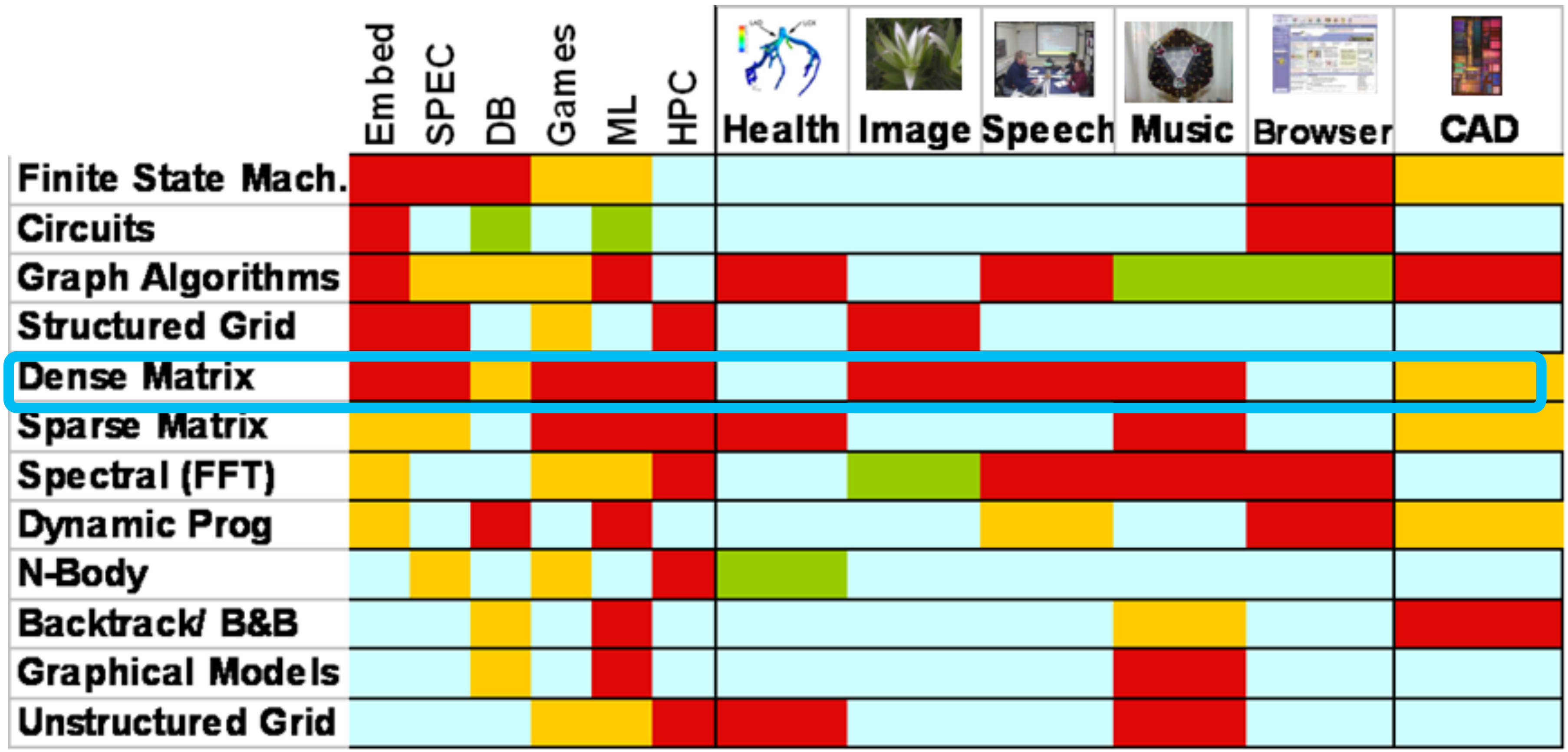
Task 4: $C_{2,2} = A_{2,1}B_{1,2} + A_{2,2}B_{2,2}$

Other task decompositions possible

# Background:
# Dense linear algebra and BLAS

# Motifs

The Motifs (formerly "Dwarfs") from "The Berkeley View" (Asanovic et al.) form key computational patterns




The Landscape of Parallel Computing Research: A View from Berkeley
http://www.eecs.berkeley.edu/Pubs/TechRpts/2006/EECS-2006-183.pdf

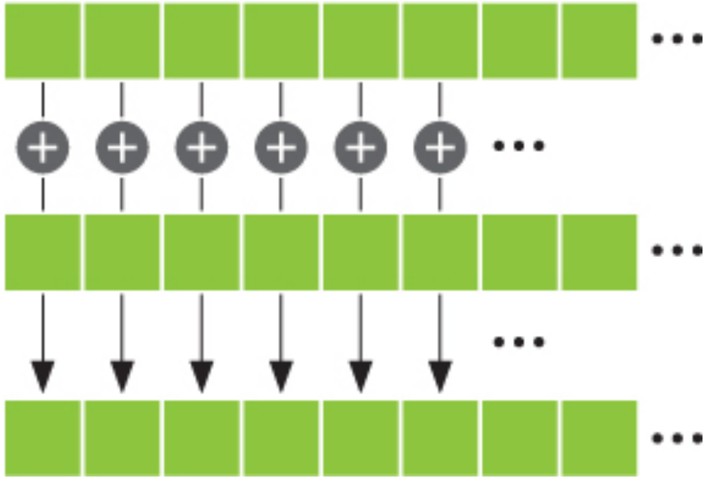
# Dense linear algebra

- Software library solving linear system

- BLAS (Basic Linear Algebra Subprogram)
  - Vector, matrix vector, matrix matrix
- Linear Systems: $Ax=b$
- Least Squares: choose x to minimize $||Ax-b||_2$
  - Overdetermined or underdetermined
  - Unconstrained, constrained, weighted
- Eigenvalues and vectors of Symmetric Matrices
  - Standard ($Ax = \lambda x$), Generalized ($Ax=\lambda Bx$)
- Eigenvalues and vectors of Unsymmetric matrices
  - Eigenvalues, Schur form, eigenvectors, invariant subspaces
  - Standard, Generalized
- Singular Values and vectors (SVD)
  - Standard, Generalized
- Different matrix structures
  - Real, complex; Symmetric, Hermitian, positive definite; dense, triangular, banded …
- Level of detail
  - Simple Driver
  - Expert Drivers with error bounds, extra-precision, other options
  - Lower level routines ("apply certain kind of orthogonal transformation", matmul…)

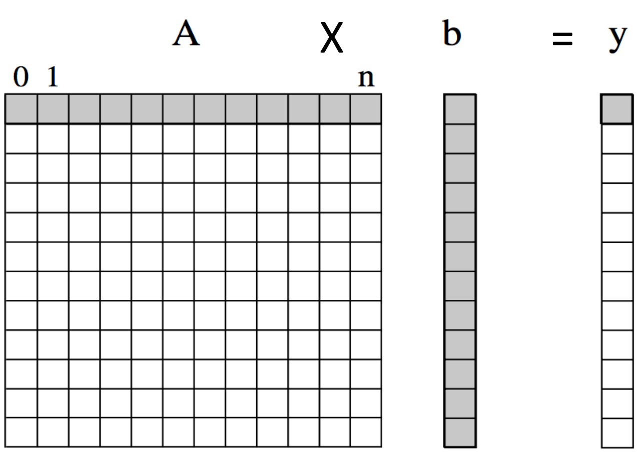
# BLAS (Basic Linear Algebra Subprogram)

- BLAS 1, 1973-1977
  - 15 operations (mostly) on vectors (1-d array)
    - "AXPY" ( $y = \alpha \cdot x + y$ ), dot product, scale ( $x = \alpha \cdot x$ )
  - Up to 4 versions of each (S/D/C/Z), 46 routines, 3300 LOC
  - Why BLAS 1 ? They do $O(n^1)$ ops on $O(n^1)$ data
  - AXPY ( $y = \alpha \cdot x + y$ )
    - 2n flops on 3n read/writes
    - Computational intensity = (2n)/(3n) = 2/3



# BLAS 2

- BLAS 2, 1984-1986
  - 25 operations (mostly) on matrix/vector pairs
  - "GEMV": y = α·A·x + β·x, "GER": A = A + α·x·yT, x = T-1·x
  - Up to 4 versions of each (S/D/C/Z), 66 routines, 18K LOC
- Why BLAS 2 ? They do $O(n^2)$ ops on $O(n^2)$ data
  - Computational intensity still just $\sim(2n^2)/(n^2) = 2$

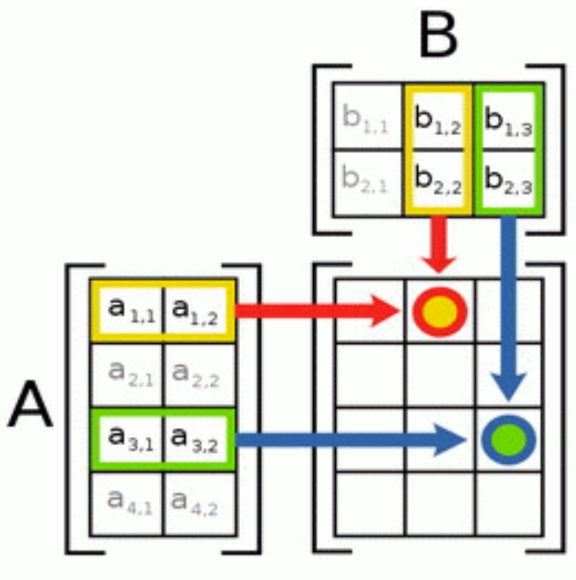


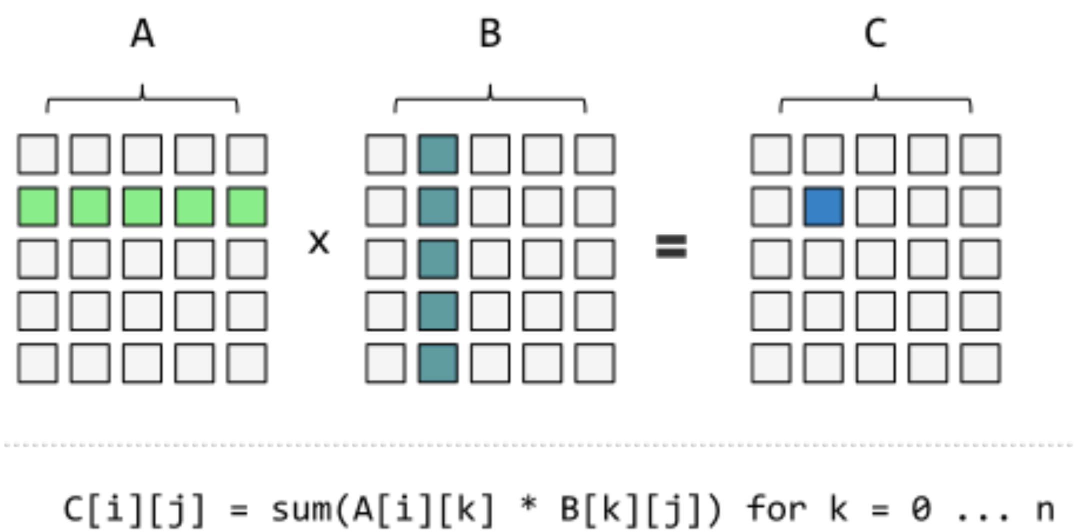

$$A\mathbf{x} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \end{bmatrix}$$

# BLAS 3

- BLAS 3, 1987-1988
  - 9 operations (mostly) on matrix/matrix pairs
    - "GEMM": C = α·A·B + β·C, C = α·A·AT + β·C,  B = T-1·B
  - Up to 4 versions of each (S/D/C/Z), 30 routines, 10K LOC
  - Why BLAS 3 ?  They do $O(n^3)$ ops on $O(n^2)$ data
    - Computational intensity $(2n^3)/(4n^2) = n/2$ – big at last!
    - Good for machines with caches, deep mem hierarchy

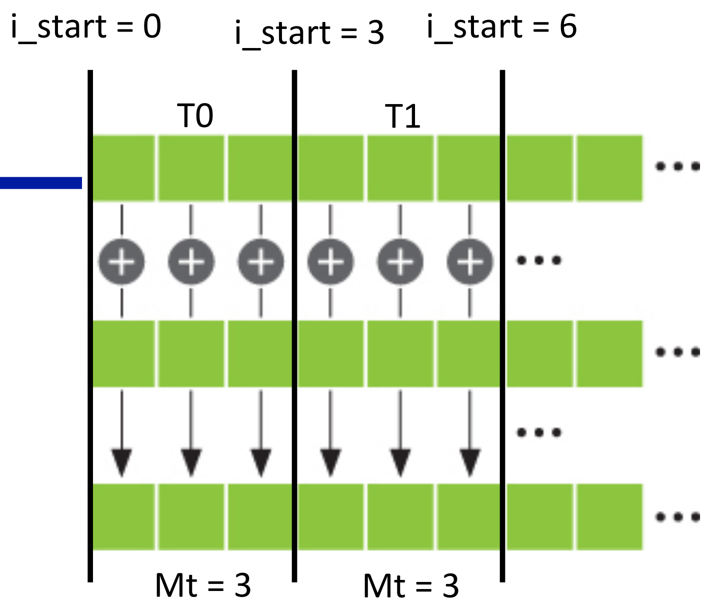
A[M][K] * B[K][N] = C[M][N]



## Practice: AXPY, Matrix Vector, and Matrix Multiplication

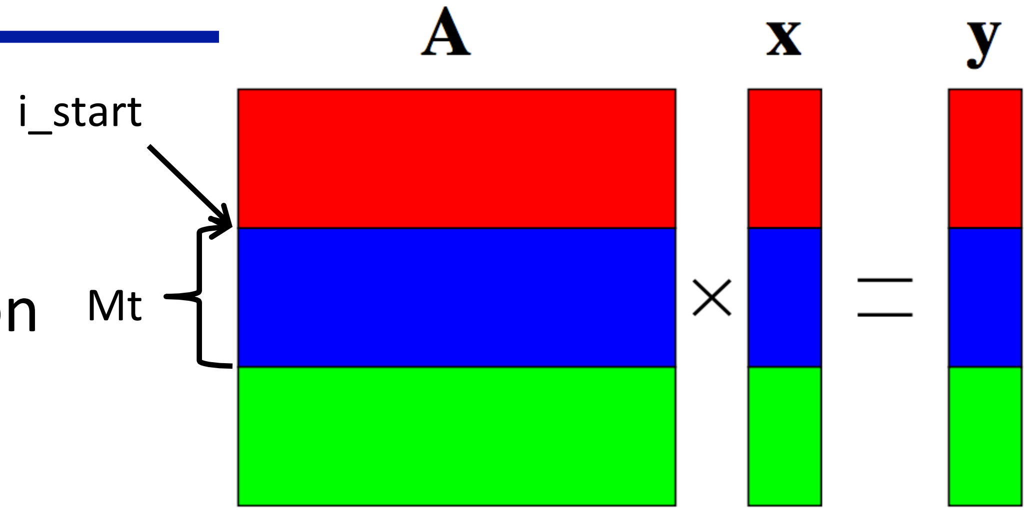
# BLAS 1: AXPY




- y = α·x + y
  - x and y are vectors of size N
    - In C, x[N], y[N]
  - α is scalar
- Decomposition is simple
  - Terms: partition, distribution, the same
  - Evenly divide N by num_tasks
    - Handle corner cases, non divisible of N by num_tasks

# BLAS 2: Matrix Vector Multiplication

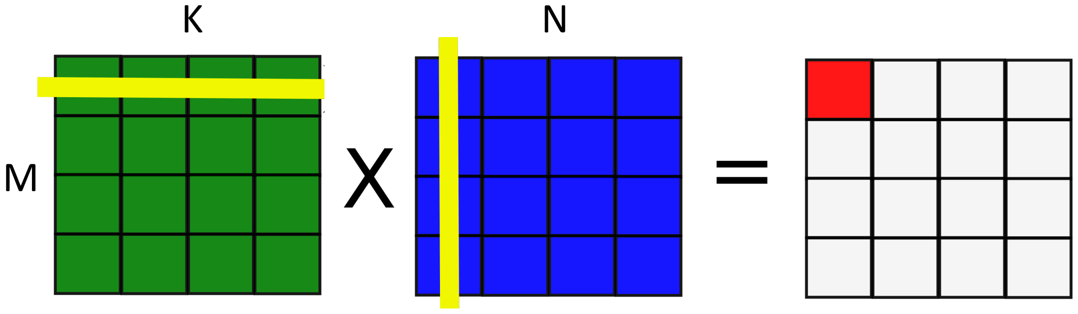
- y = A·x
  - A[M][N], x[N], y[N]
- Row-wise decomposition



# BLAS 3: Dense Matrix Multiplication

**A[M][K] * B[k][N] = C[M][N]**

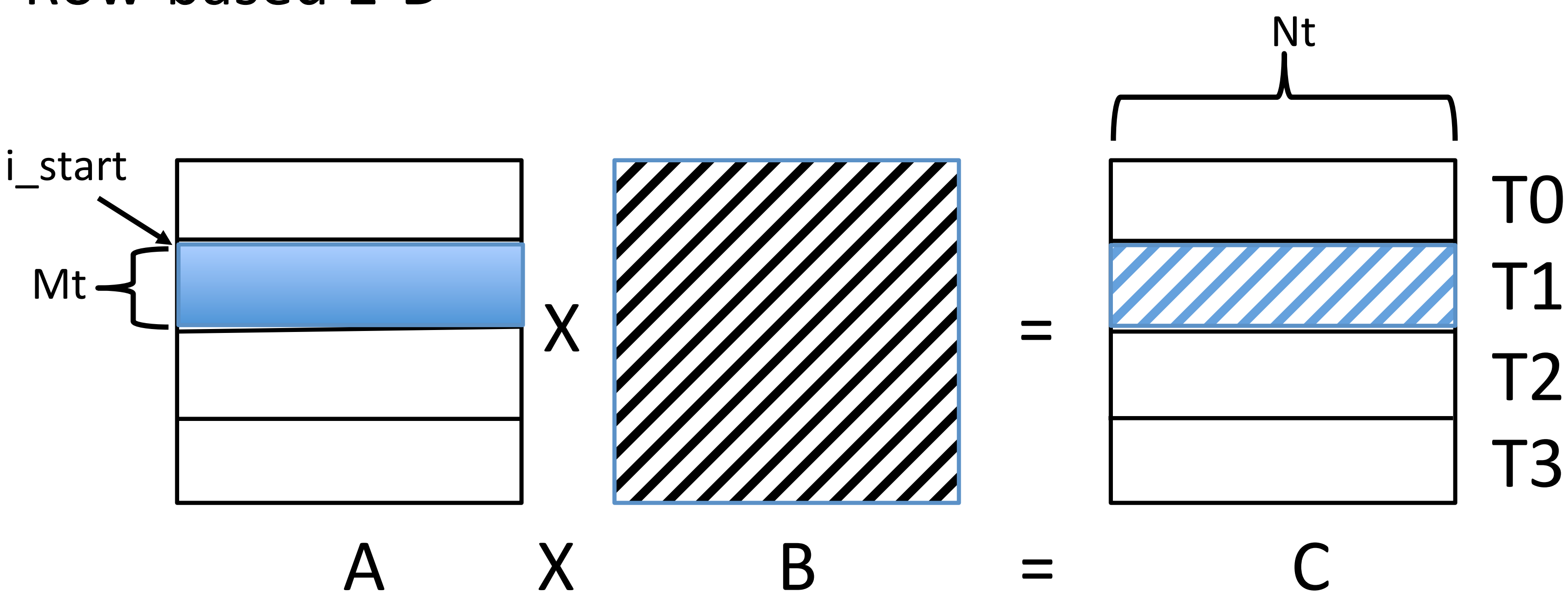
- Base
- Base_1: column major order of access
- row1D_dist
- column1D_dist
- rowcol2D_dist




- Decomposition is to calculate Mt and Nt

# BLAS 3: Dense Matrix Multiplication

- Row-based 1-D

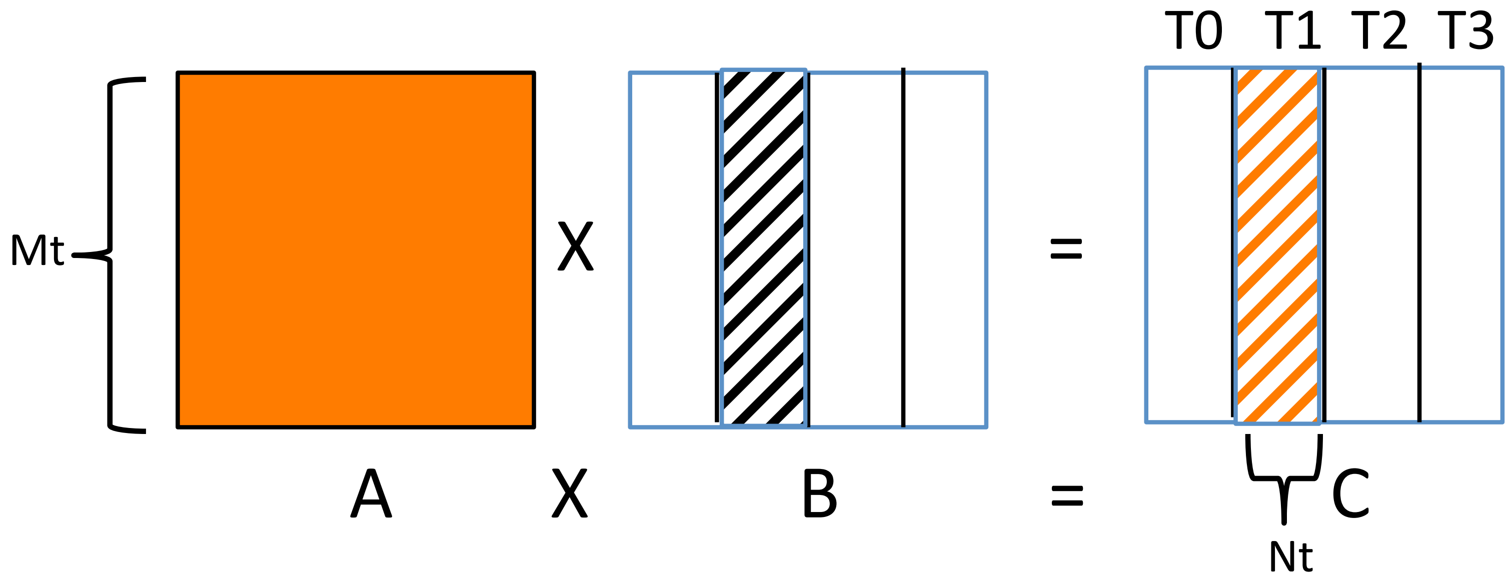



Mt = N/num_tasks
i_start = tid * Mt;

Nt = N
j_start = 0

# BLAS 3: Dense Matrix Multiplication

- Column-based 1-D

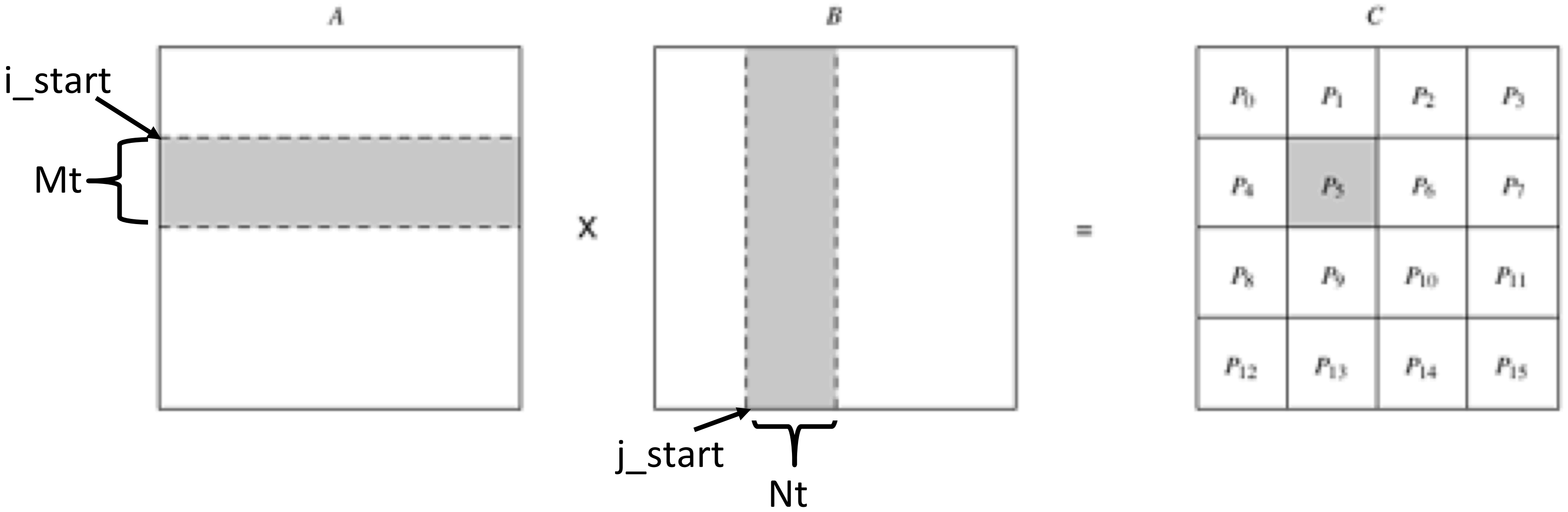



i_start = 0
Mt = N

Nt = N/num_tasks
j_start = tid * Nt

# BLAS 3: Dense Matrix Multiplication

- Row/Column-based 2-D




- If you do nested parallelism
  - export OMP_NESTED=true

# Submatrix Multiplication

- Work with any of the three decomposition

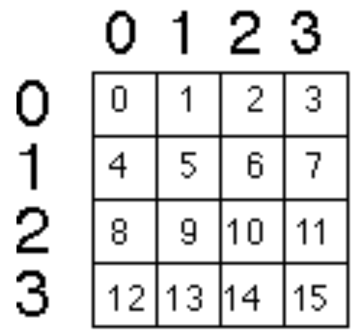
```
/* compute submatrix multiplication, A[start:length] notation
 * A[i_start:Mt][N] x B[N][j_start:Nt] = C[i_start:Mt][j_start:Nt]
 */
void matmul_base_sub(int i_start, int j_start, int Mt, int Nt, int N,
        REAL A[][N], REAL B[][N], REAL C[][N]) {
        int i, j, k;
        for (i = i_start; i < Mt+i_start; i++) {
        for (j = j_start; j < Nt + j_start; j++) {
            C[i][j] = 0;
            for (k = 0; k < N; k++)
                C[i][j] += A[i][k]*B[k][j];
        }
    }
}
```

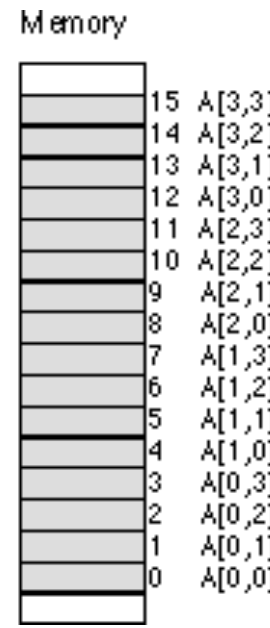
# Background:
# C multidimensional array

# Vector/Matrix and Array in C

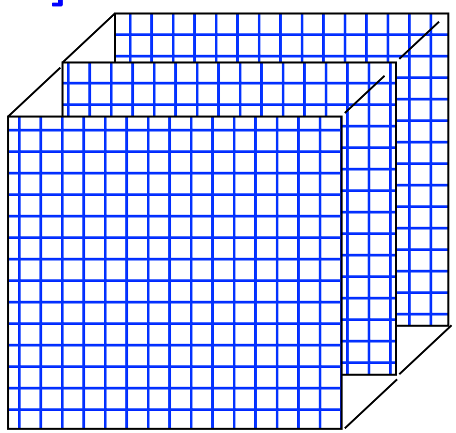
- C has row-major storage for multiple dimensional array
  - A[2,2] is followed by A[2,3]

char A[4][4]



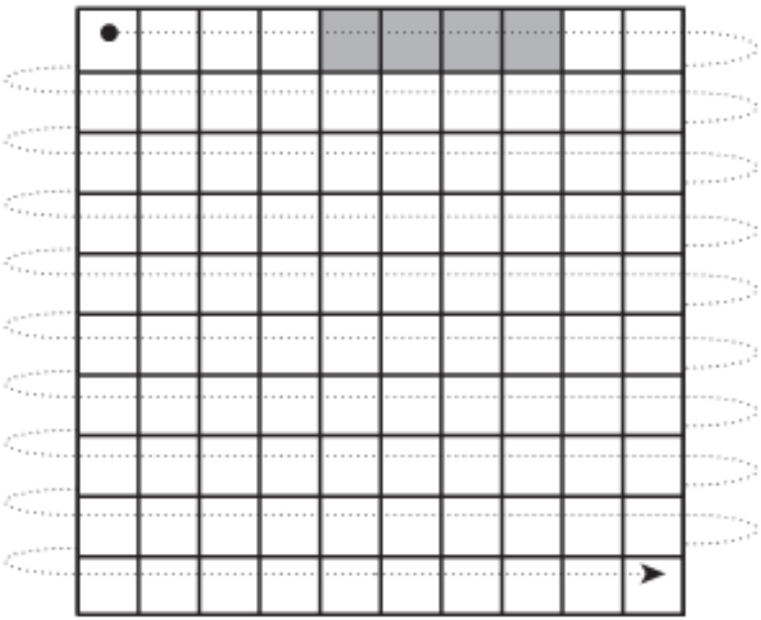

- 3-dimensional array
  - B[3][100][100]



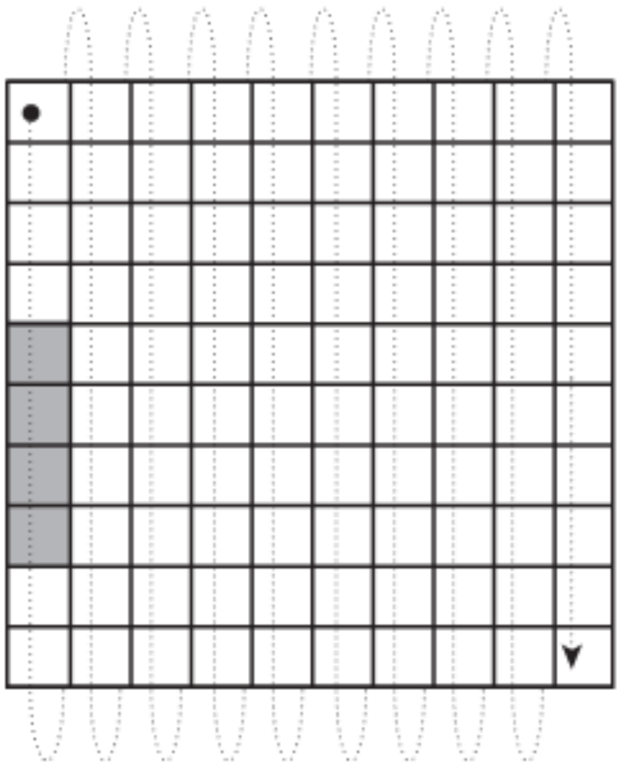
- Think it as recursive definition
  - A[4][10][32]

# Column Major

**Fortran is column major**



Row-major order



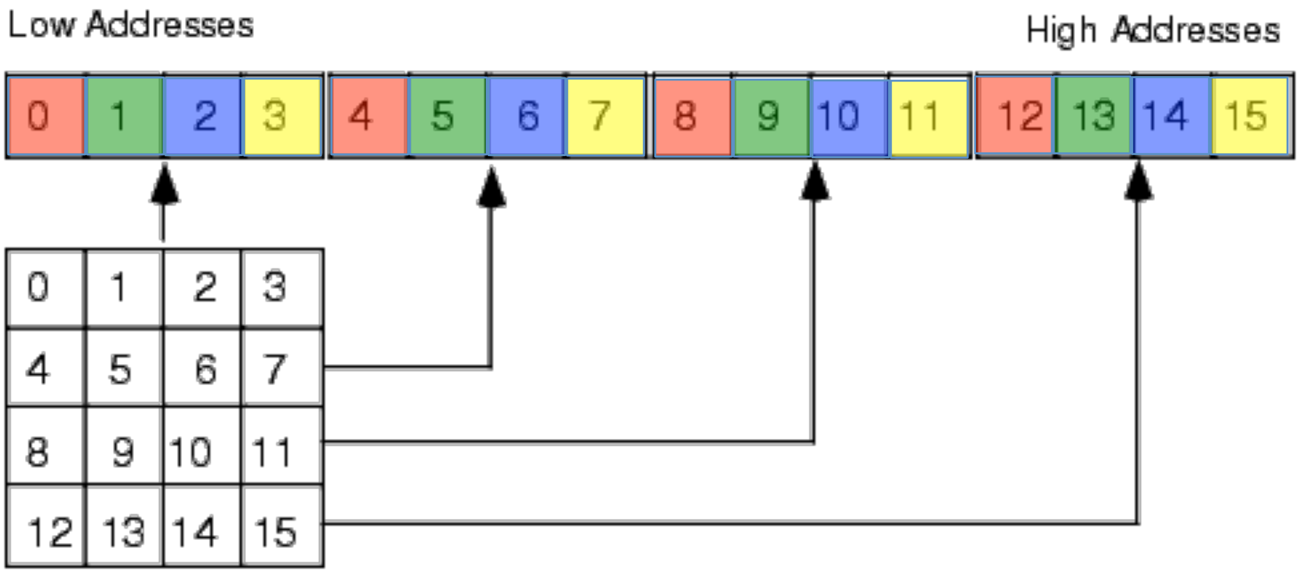
Column-major order

# Array Layout: Why We Care?

## 1. Makes a big difference for access speed

- For performance, set up code to go in row major order in C
  - Caching: each read from memory will bring other adjacent elements to the cache line
- (Bad) Example: 4 vs 16 accesses
  - matmul_base_1

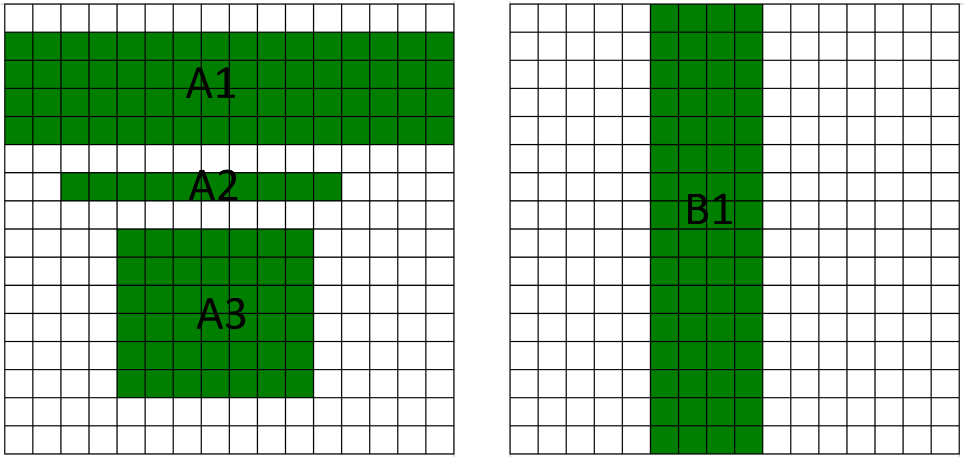
```
for i = 1 to n
      for j = 1 to n
            A[j][i] = value
```



# Array Layout: Why We Care?

## 2. Affect decomposition and data movement

- Decomposition may create submatrices that are in non-contiguous memory locations, e.g. A3 and B1
- Submatrices in contiguous memory location of 2-D row major matrix
  - A single-row submatrix, e.g. A2
  - A submatrix formed with adjacent rows with full column length, e.g. A1

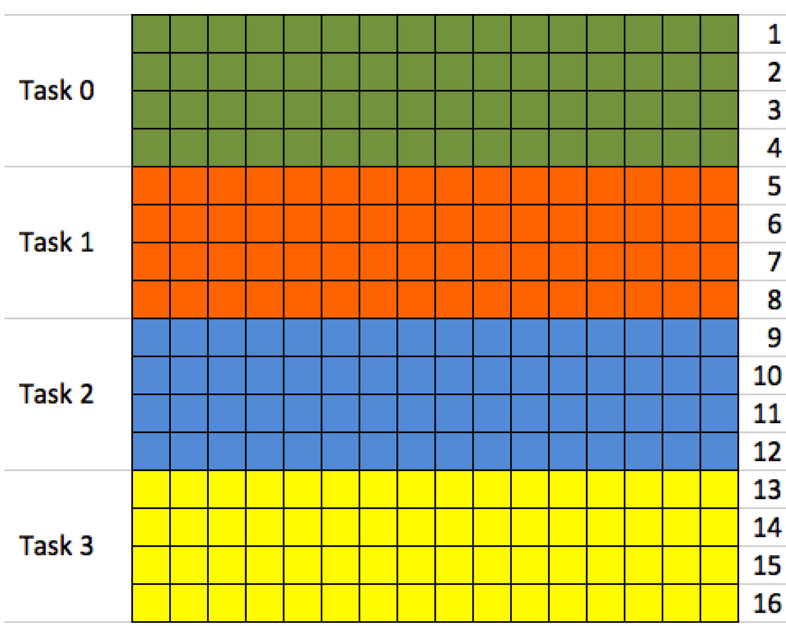


# Array Layout: Why We Care?

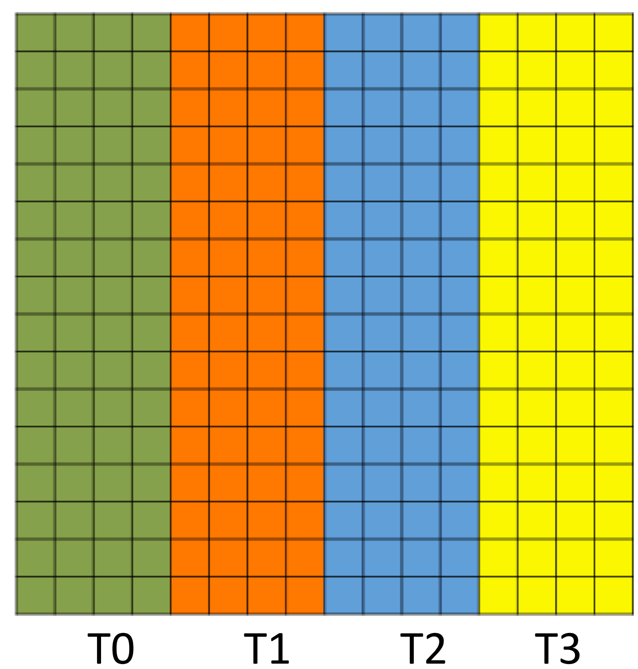
## 2. Affect decomposition and submatrix

- Row or column wise distribution of 2-D row-major array
- # of data movement to exchange data between T0 and T1
  – Row-wise: one memory copy by each
  – Column-wise: 16 copies each

Row-wise distribution




Column-wise distribution



# Array and pointers in C

- In C, an array is a pointer + dimensionality
  – They are literally the same in binary, i.e. pointer to the first element, referenced as base address
- Cast and assignment from array to pointe, int A[M][N]
  - A, &A[0][0], and A[0] have the same value, i.e. the pointer to the first element of the array
- Cast a pointer to an array
  – int *ap; int (*A)[N] = (int(*)[N])ap; A[i][j] ....
- Address calculation for array references
  – Address of A[i][j] = A + (i*N+j)*sizeof (int)