## Lecture: Manycore GPU Architectures and Programming, Part 2

### **CSCE 569 Parallel Computing**

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### Manycore GPU Architectures and Programming: Outline

- Introduction
  - GPU architectures, GPGPUs, and CUDA
- GPU Execution model
- CUDA Programming model
  - Working with Memory in CUDA
    - Global memory, shared and constant memory
  - Streams and concurrency
  - CUDA instruction intrinsic and library
  - Performance, profiling, debugging, and error handling
  - Directive-based high-level programming model
    - OpenACC and OpenMP

## How is the GPU controlled?

- The CUDA API is split into:
  - The CUDA Management API
  - The CUDA Kernel API
- The CUDA Management API is for a variety of operations
  - GPU memory allocation, data transfer, execution, resource creation
  - Mostly regular C function and calls
- The CUDA Kernel API is used to define the computation to be performed by the GPU
  - C extensions

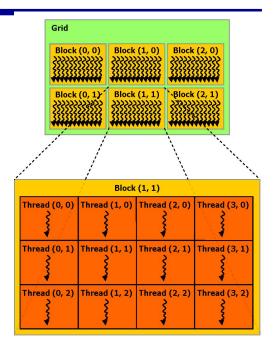
## How is the GPU controlled?

- A CUDA kernel:
  - Defines the operations to be **performed by a single thread** on the GPU
  - Just as a C/C++ function defines work to be done on the CPU
  - Syntactically, a kernel looks like C/C++ with some extensions

```
global void kernel(...) {
   ...
}
```

- Every CUDA thread executes the same kernel logic (SIMT)
- Initially, the only difference between threads are their *thread coordinates*

- CUDA thread hierarchy
  - Warp = SIMT Group
  - Thread Block = SIMT Groups that run concurrently on an SM
  - Grid = All Thread Blocks created by the same kernel launch



- Launching a kernel is simple and similar to a function call.
  - kernel name and arguments
  - # of thread blocks/grid and # of threads/block to create:

kernel<<<nblocks,</pre>

threads\_per\_block>>>(arg1, arg2, ...);

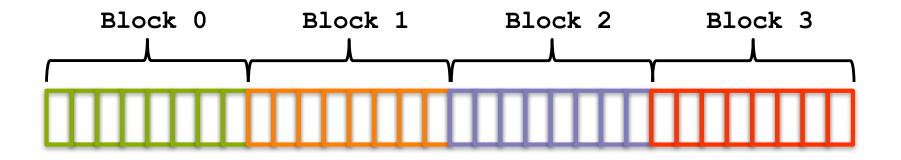
- In CUDA, only thread blocks and grids are first-class citizens of the programming model.
- The number of warps created and their organization are implicitly controlled by the *kernel launch configuration*, but never set explicitly.

- GPU threads can be configured in one-, two-, or threedimensional layouts
  - One-dimensional blocks and grids:

int nblocks = 4;

int threads\_per\_block = 8;

kernel<<<nblocks, threads\_per\_block>>>(...);

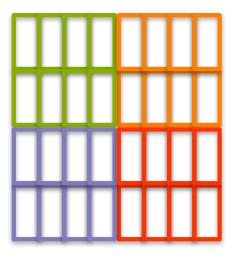


- GPU threads can be configured in one-, two-, or threedimensional layouts
  - Two-dimensional blocks and grids:

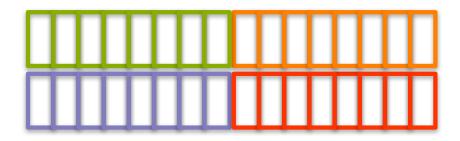
```
dim3 nblocks(2,2)
```

```
dim3 threads_per_block(4,2);
```

```
kernel<<<nblocks, threads_per_block>>>(...);
```



- GPU threads can be configured in one-, two-, or threedimensional layouts
  - Two-dimensional grid and one-dimensional blocks: dim3 nblocks(2,2); int threads\_per\_block = 8; kernel<<<nblocks, threads\_per\_block>>>(...);

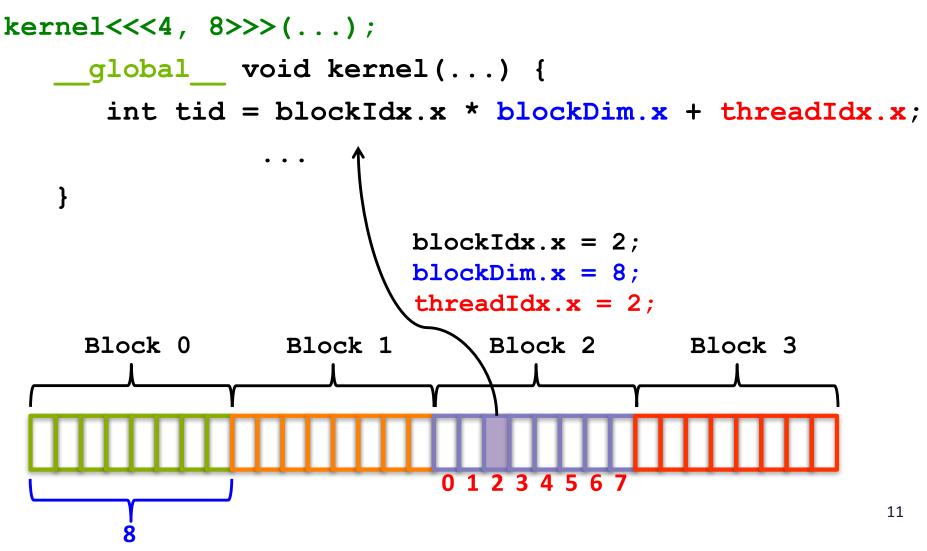


- On the GPU, the number of blocks and threads per block is exposed through intrinsic thread coordinate variables:
  - Dimensions

– IDs

Variable	Meaning
gridDim.x, gridDim.y, gridDim.z	Number of blocks in a kernel launch.
<pre>blockIdx.x, blockIdx.y,</pre>	Unique ID of the block that contains the current thread.
blockDim.x, blockDim.y, blockDim.z	Number of threads in each block.
<pre>threadIdx.x, threadIdx.y,     threadIdx.z</pre>	Unique ID of the current thread within its block.

to calculate a **globally unique ID** for a thread on the GPU inside a one-dimensional grid and one-dimensional block:



- Thread coordinates offer a way to differentiate threads and identify thread-specific input data or code paths.
  - Link data and computation, a mapping

```
__global___void kernel(int *arr) {
    int tid = blockIdx.x * blockDim.x + threadIdx.x;
    if (tid < 32) {
        arr[tid] = f(arr[tid]);
    } code path for threads with tid < 32
    } else {
        arr[tid] = g(arr[tid]);
    }
}</pre>
```

Thread Divergence: recall that useless code path is executed, but then disabled in SIMT execution model

### How is GPU memory managed?

- CUDA Memory Management API
  - Allocation of GPU memory
  - Transfer of data from the host to GPU memory
  - Free-ing GPU memory
  - Foo(int A[][N]) { }

Host Function	CUDA Analogue
malloc	cudaMalloc
memcpy	cudaMemcpy
free	cudaFree

 Allocate size bytes of GPU memory and store their address at \*devPtr

cudaError\_t cudaFree(void \*devPtr);

- Release the device memory allocation stored at devPtr
- Must be an allocation that was created using cudaMalloc

### How is GPU memory managed?

#### cudaError\_t cudaMemcpy(

void \*dst, const void \*src, size\_t count,

enum cudaMemcpyKind kind);

- Transfers count bytes from the memory pointed to by src to dst
- kind can be:
  - cudaMemcpyHostToHost,
  - cudaMemcpyHostToDevice,
  - cudaMemcpyDeviceToHost,
  - cudaMemcpyDeviceToDevice
- The locations of dst and src must match kind, e.g. if kind is cudaMemcpyHostToDevice then src must be a host array and dst must be a device array

### How is GPU memory managed?

void \*d\_arr, \*h\_arr; h\_addr = ... ; /\* init host memory and data \*/ // Allocate memory on GPU and its address is in d\_arr cudaMalloc((void \*\*)&d\_arr, nbytes);

// Transfer data from a device to a host
cudaMemcpy(h\_arr, d\_arr, nbytes,

cudaMemcpyDeviceToHost);

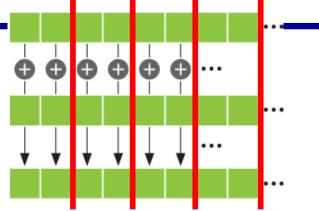
```
// Free the allocated memory
cudaFree(d_arr);
```

## **CUDA Program Flow**

- At its most basic, the flow of a CUDA program is as follows:
  - 1. Allocate GPU memory
  - 2. Populate GPU memory with inputs from the host
  - 3. Execute a GPU kernel on those inputs
  - 4. Transfer outputs from the GPU back to the host
  - 5. Free GPU memory
- Let's take a look at a simple example that manipulates data

### **AXPY Example with OpenMP: Multicore**

- $y = \alpha \cdot x + y$ 
  - x and y are vectors of size n
  - $\alpha$  is scalar

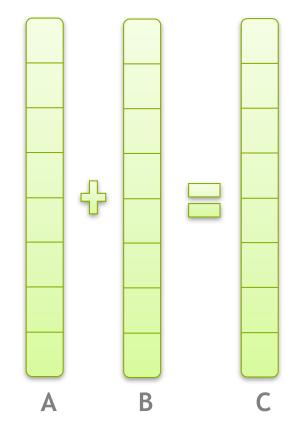


1	<pre>void axpy(REAL *x, REAL *y, long n, REAL a) {</pre>
2	<pre>#pragma omp parallel for shared(x, y, n, a)</pre>
3	<b>for (int</b> i = 0; i < n; ++i)
4	y[i] += a * x[i];
5	}

- Data (x, y and a) are shared
  - Parallelization is relatively easy

### **CUDA Program Flow**

- AXPY is an **embarrassingly parallel problem** 
  - How can vector addition be parallelized?
  - How can we map this to GPUs?
- Each thread does one element



### **AXPY Offloading To a GPU using CUDA**

```
CUDA kernel. Each thread takes care of one element of c
 2
     global void axpy(REAL *x, REAL *y, int n, REAL a) {
 3
       int id = blockIdx.x*blockDim.x+threadIdx.x:
                                                                           0
       if (id < n) y[id] += a * x[id];
 4
 5
 6
   int main( int argc, char* argv[] ) {
 7
 8
 9
       // ... init host a, x and y
       // Allocate memory for each vector on GPU
10
                                                             Memory allocation on device
11
       cudaMalloc(&d x, size);
12
       cudaMalloc(&d_y, size);
13
14
       // Copy host vectors to device
15
       cudaMemcpy( d_x, h_x, size, cudaMemcpyHostToDevice);
                                                                Memcpy from host to device
       cudaMemcpy( d y, h y, size, cudaMemcpyHostToDevice);
16
17
       int blockSize, gridSize;
18
19
       blockSize = 1024;
                                                                 Launch parallel execution
20
       gridSize = (int)ceil((float)n/blockSize);
21
       axpy<<<qridSize, blockSize>>>(d_x, d_y, n, a);
22
23
       // Copy array back to host
                                                                 Memcpy from device to host
24
       cudaMemcpy( h y, d y, size, cudaMemcpyDeviceToHost
                                                            )
25
26
       // Release device memory
       cudaFree(d_x);
27
                                                                 Deallocation of dev memory
       cudaFree(d y);
28
                                                                                     20
29 }
```

## **CUDA Program Flow**

- Consider the workflow of the example vector addition vecAdd.cu:
  - **1.** Allocate space for A, B, and C on the GPU
  - 2. Transfer the initial contents of A and B to the GPU
  - 3. Execute a kernel in which each thread sums  $A_{i}$  and  $B_{i}$ , and stores the result in  $C_{i}$
  - 4. Transfer the final contents of C back to the host
  - 5. Free A, B, and C on the GPU

#### Modify to C = A+B+C

#### A = B\*C;

#### we will need both C and A in the host side after GPU

#### computation.

- Compile and running on bridges:
  - https://passlab.github.io/CSCE569/resources/HardwareSoftware.html#inte ractive
  - copy gpu\_code\_examples folder from my home folder
    - cp –r ~yan/gpu\_code\_examples ~
  - \$nvcc -Xcompiler -fopenmp vectorAdd.cu
  - \$./a.out

### **More Examples and Exercises**

- Matvec:
  - Version 1: each thread computes one element of the final vector
  - Version 2:
- Matmul in assignment #4
  - Version 1: each thread computes one row of the final matrix C

## **CUDA SDK Examples**

- CUDA Programming Manual:
  - <u>http://docs.nvidia.com/cuda/cuda-c-programming-guide</u>
- CUDA SDK Examples on bridges
  - module load gcc/5.3.0 cuda/8.0
  - export CUDA\_PATH=/opt/packages/cuda/8.0
  - /opt/packages/cuda/8.0/samples
- Copy to your home folder
  - cp –r /opt/packages/cuda/8.0/samples ~/CUDA\_samples
- Do a "make" in the folder, and it will build all the sources
- Or go to a specific example folder and make, it will build only the binary
- Find ones you are interested in and run to see

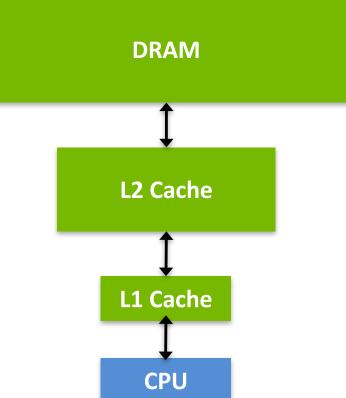
### **Inspecting CUDA Programs**

- Debugging CUDA program:
  - cuda-gdb debugging tool, like gdb
- Profiling a program to examine the performance
  - Nvprof tool, like gprof
  - Nvprof ./vecAdd

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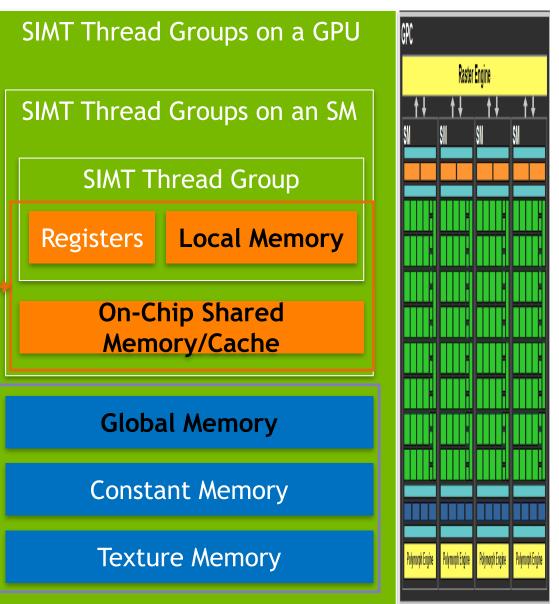
- A memory hierarchy emulates a large amount of lowlatency memory
  - Cache data from a large, high-latency memory bank in a small low-latency memory bank



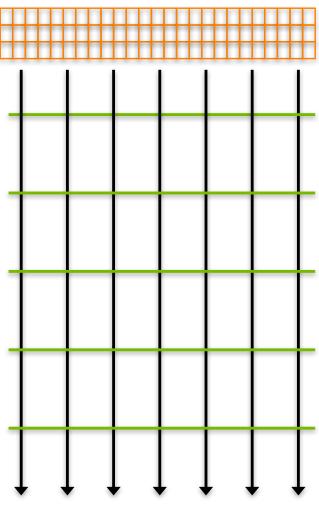
**CPU Memory Hierarchy** 

## **GPU Memory Hierarchy**

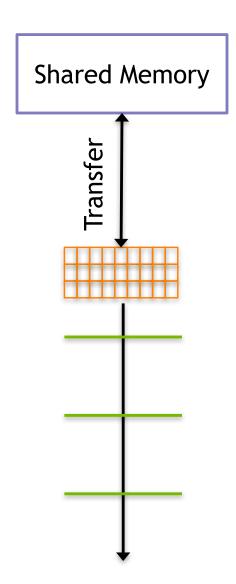
- More complex than the CPU memory
  - Many different types of memory, each with special-purpose characteristics
    - SRAM-
    - DRAM 、
  - More *explicit* control over data movement



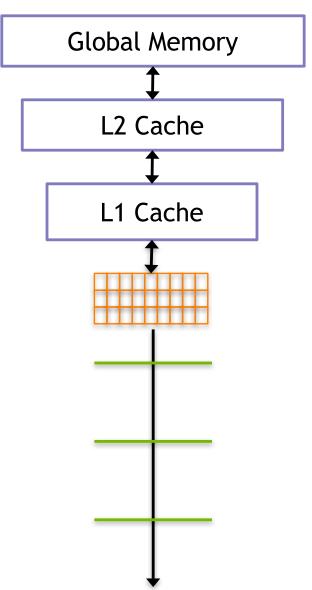
- Registers (SRAM)
  - Lowest latency memory space on the GPU
  - Private to each CUDA thread
  - Constant pool of registers per-SM divided among threads in resident thread blocks
  - Architecture-dependent limit on number of registers per thread
  - Registers are not explicitly used by the programmer, implicitly allocated by the compiler
  - maxrregcount compiler option allows you to limit # registers per thread



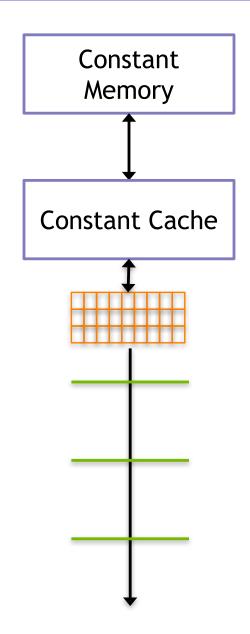
- Shared Memory (SRAM)
  - Declared with the \_\_shared\_ keyword
  - Low-latency, high bandwidth
  - Shared by all threads in a thread block
  - Explicitly allocated and managed by the programmer, manual L1 cache
  - Stored on-SM, same physical memory as the GPU L1 cache
  - On-SM memory is statically partitioned between L1 cache and shared memory



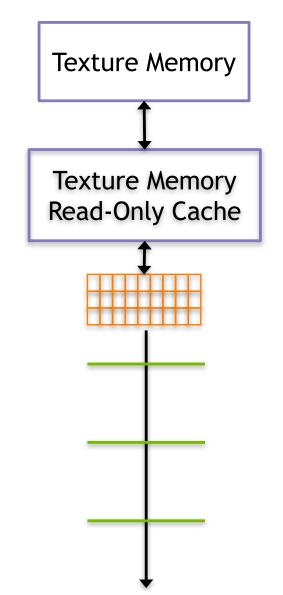
- GPU Caches (SRAM)
  - Behaviour of GPU caches is architecture-dependent
  - Per-SM L1 cache stored on-chip
  - Per-GPU L2 cache stored off-chip, caches values for all SMs
  - Due to parallelism of accesses, GPU caches do not follow the same LRU rules as CPU caches



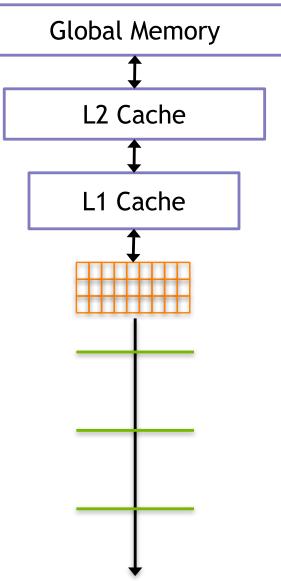
- Constant Memory (DRAM)
  - Declared with the \_\_\_\_constant\_ keyword
  - Read-only
  - Limited in size: 64KB
  - Stored in device memory (same physical location as Global Memory)
  - Cached in a per-SM constant cache
  - Optimized for all threads in a warp accessing the same memory cell



- Texture Memory (DRAM)
  - Read-only
  - Stored in device memory (same physical location as Global Memory)
  - Cached in a texture-only on-SM cache
  - Optimized for 2D spatial locality (caches commonly only optimized for 1D locality)
  - Explicitly used by the programmer
  - Special-purpose memory



- Global Memory (DRAM)
  - Large, high-latency memory
  - Stored in device memory (along with constant and texture memory)
  - Can be declared statically with device
  - Can be allocated dynamically with cudaMalloc
  - Explicitly managed by the programmer
  - Optimized for all threads in a warp accessing neighbouring memory cells



MEMORY	ON/OFF CHIP	CACHED	ACCESS	SCOPE	LIFETIME
Register	On	n/a	R/W	1 thread	Thread
Local	Off	†	R/W	1 thread	Thread
Shared	On	n/a	R/W	All threads in block	Block
Global	Off	†	R/W	All threads + host	Host allocation
Constant	Off	Yes	R	All threads + host	Host allocation
Texture	Off	Yes	R	All threads + host	Host allocation

QUALIFIER	VARIABLE NAME	MEMORY	SCOPE	LIFESPAN
	float var	Register	Thread	Thread
	float var[100]	Local	Thread	Thread
shared	float var†	Shared	Block	Block
				DIOCK
device	float var†	Global	Global	Application

### **Static Global Memory**

 Static Global Memory has a fixed size throughout execution time:

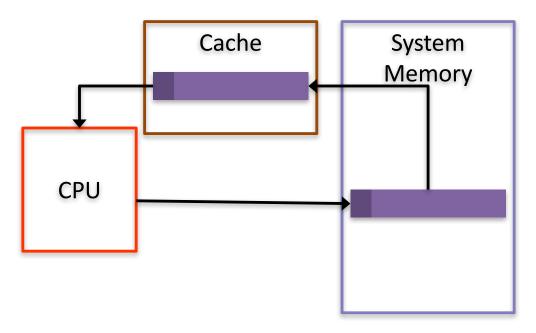
\_\_device\_\_ float devData; \_\_global\_\_ void checkGlobalVariable() printf("devData has value %f\n", devData); }

- Initialized using cudaMemcpyToSymbol:
   cudaMemcpyToSymbol(devData, &hostData, sizeof(float));
- Fetched using cudaMemcpyFromSymbol: cudaMemcpyFromSymbol(&hostData, devData, sizeof(float));

# **Dynamic Global Memory**

- We have already seen dynamic global memory
  - cudaMalloc dynamically allocates global memory
  - cudaMemcpy transfers to/from global memory
  - cudaFree frees global memory allocated by cudaMalloc
- cudaMemcpy supports 4 types of transfer:
  - cudaMemcpyHostToHost, cudaMemcpyHostToDevice, cudaMemcpyDeviceToHost, cudaMemcpyDeviceToDevice
- You can also memset global memory cudaError\_t cudaMemset(void \*devPtr, int value, size\_t count);

- CPU caches are optimized for linear, iterative memory accesses
  - Cache lines ensure that accessing one memory cell brings neighbouring memory cells into cache
  - If an application exhibits good spatial or temporal locality (which many do), later references will also hit in cache

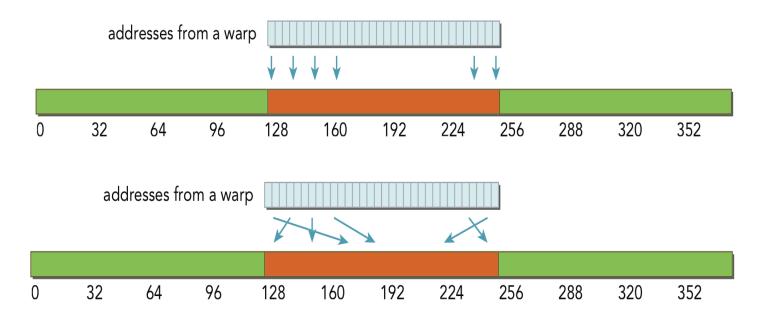


- GPU caching is a more challenging problem
  - Thousands of threads cooperating on a problem
  - Difficult to predict the next round of accesses for all threads
- For efficient global memory access, GPUs instead rely on:
  - Large device memory bandwidth
  - Aligned and coalesced memory access patterns
  - Maintaining sufficient pending I/O operations to keep the memory bus saturated and hide global memory latency

- Achieving aligned and coalesced global memory accesses is key to optimizing an application's use of global memory bandwidth
  - Coalesced: the threads within a warp reference memory addresses that can all be serviced by a single global memory transaction (think of a memory transaction as the process of bring a cache line into the cache)
  - Aligned: the global memory accesses by threads within a warp start at an address boundary that is an even multiple of the size of a global memory transaction

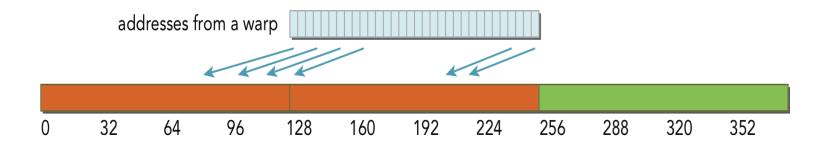
- A global memory transaction is either 32 or 128 bytes
  - The size of a memory transaction depends on which caches it passes through
  - If L1 + L2: 128 byte
  - If L2 only: 32 bytes
  - Which caches a global memory transaction passes through depends on GPU architecture and the type of access (read vs. write)

- Aligned and Coalesced Memory Access (w/L1 cache)
  - 32-thread wrap, 128-bytes memory transaction



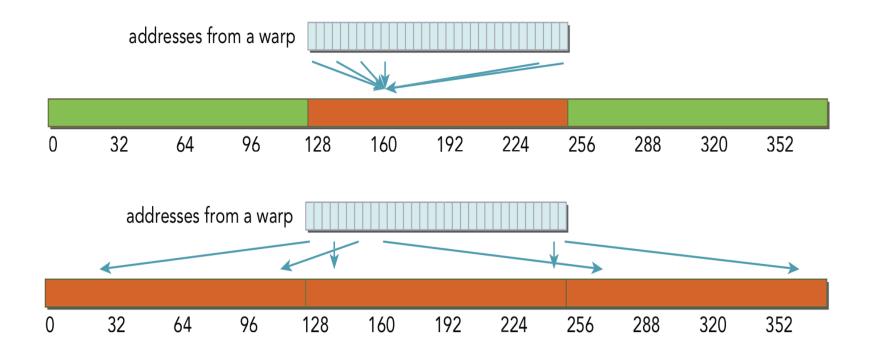
• With 128-byte access, a single transaction is required and all of the loaded bytes are used

Misaligned and Coalesced Memory Access (w/L1 cache)



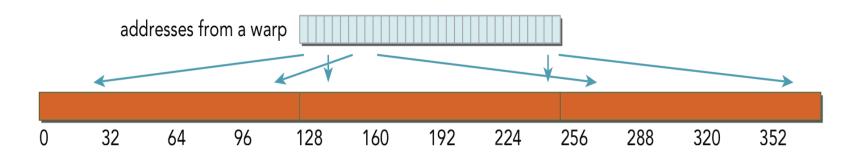
 With 128-byte access, two memory transactions are required to load all requested bytes. Only half of the loaded bytes are used.

• Misaligned and Uncoalesced Memory Access (w/L1 cache)



With uncoalesced loads, many more bytes loaded than requested

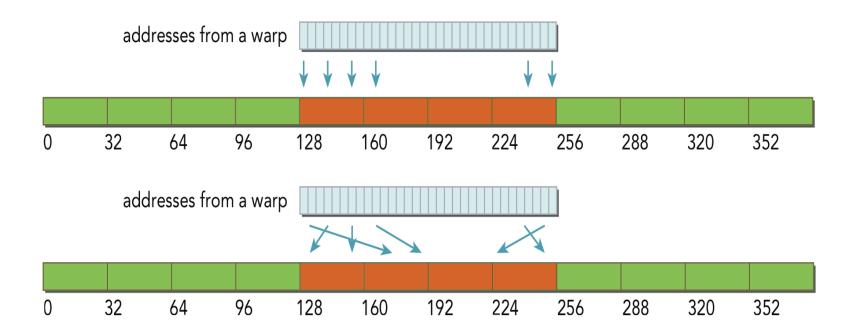
• Misaligned and Uncoalesced Memory Access (*w/L1 cache*)



 One factor to consider with uncoalesced loads: while the efficiency of this access is very low it may bring many cache lines into L1/L2 cache which are used by later memory accesses. The GPU is flexible enough to perform well, even for applications that present suboptimal access patterns.

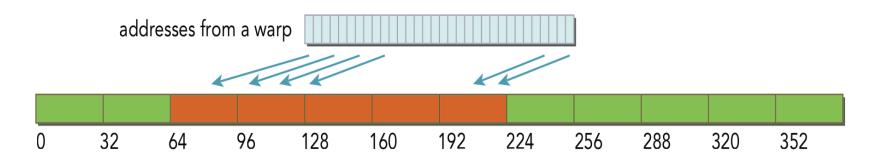
- Memory accesses that are not cached in L1 cache are serviced by 32-byte transactions
  - This can improve memory bandwidth utilization
  - However, the L2 cache is device-wide, higher latency than L1, and still relatively small → many applications may take a performance hit if L1 cache is not used for reads

• Aligned and Coalesced Memory Access (*w/o L1 cache*)



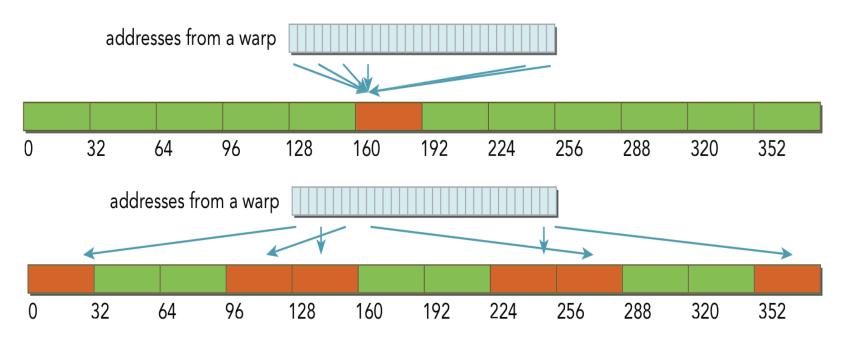
• With 32-byte transactions, four transactions are required and all of the loaded bytes are used

Misaligned and Coalesced Memory Access (w/o L1 cache)



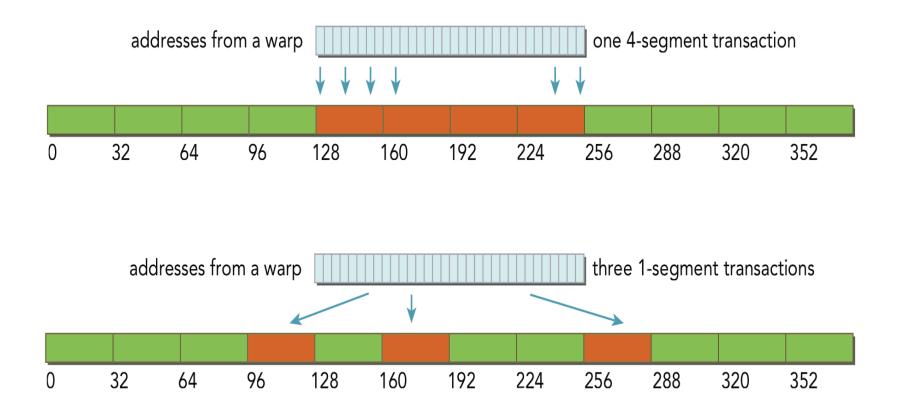
 With 32-byte transactions, extra memory transactions are still required to load all requested bytes but the number of wasted bytes is likely reduced, compared to 128-byte transactions.

Misaligned and Uncoalesced Memory Access (w/o L1 cache)



• With uncoalesced loads, more bytes loaded than requested but better efficiency than with 128-byte transactions

Global Memory Writes are always serviced by 32-byte transactions

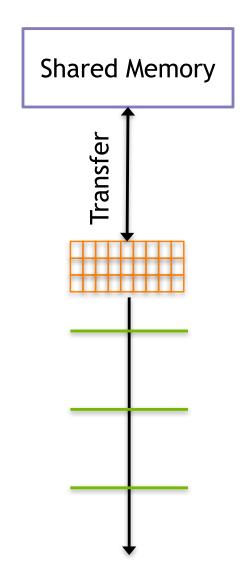


### **Global Memory and Special-Purpose Memory**

- Global memory is widely useful and as easy to use as CPU DRAM
- Limitations
  - Easy to find applications with memory access patterns that are intrinsically poor for global memory
  - Many threads accessing the same memory cell → poor global memory efficiency
  - Many threads accessing sparse memory cells → poor global memory efficiency
- Special-purpose memory spaces to address these deficiencies in global memory
  - Specialized for different types of data, different access patterns
  - Give more control over data movement and data placement than CPU architectures do

# **Shared Memory**

- Declared with the \_\_\_\_shared\_\_\_
   keyword
- Low-latency, high bandwidth
- Shared by all threads in a thread block
- Explicitly allocated and managed by the programmer, manual L1 cache
- Stored on-SM, same physical memory as the GPU L1 cache
- On-SM memory is statically partitioned between L1 cache and shared memory



# **Shared Memory Allocation**

- Shared memory can be allocated statically or dynamically
- Statically Allocated Shared Memory
  - Size is fixed at compile-time
  - Can declare many statically allocated shared memory variables
  - Can be declared globally or inside a device function
  - Can be multi-dimensional arrays

#### \_\_\_\_shared\_\_\_\_int s\_arr[256][256];

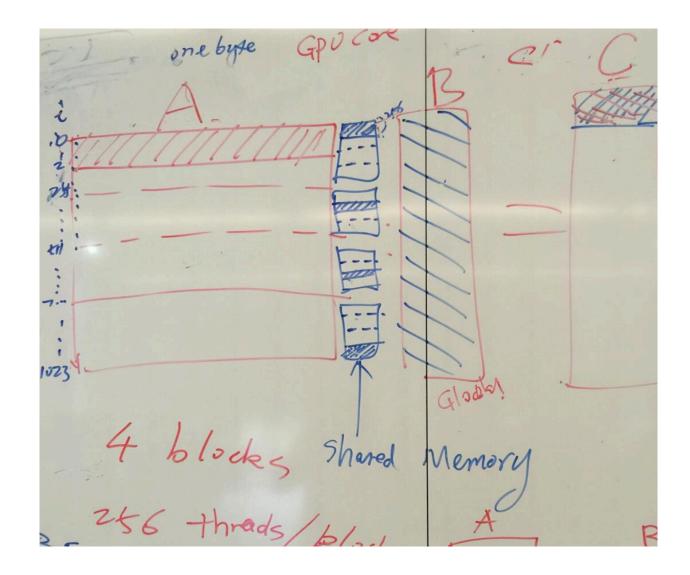
# **Shared Memory Allocation**

- Dynamically Allocated Shared Memory
  - Size in bytes is set at kernel launch with a third kernel launch configurable
  - Can only have one dynamically allocated shared memory array per kernel
  - Must be one-dimensional arrays

```
__global___ void kernel(...) {
    extern __shared__ int s_arr[];
    ....
}
```

```
kernel<<<nblocks, threads_per_block,
shared_memory_bytes>>>(...);
```

### Matvec using shared memory



### **Matrix Vector Multiplication**

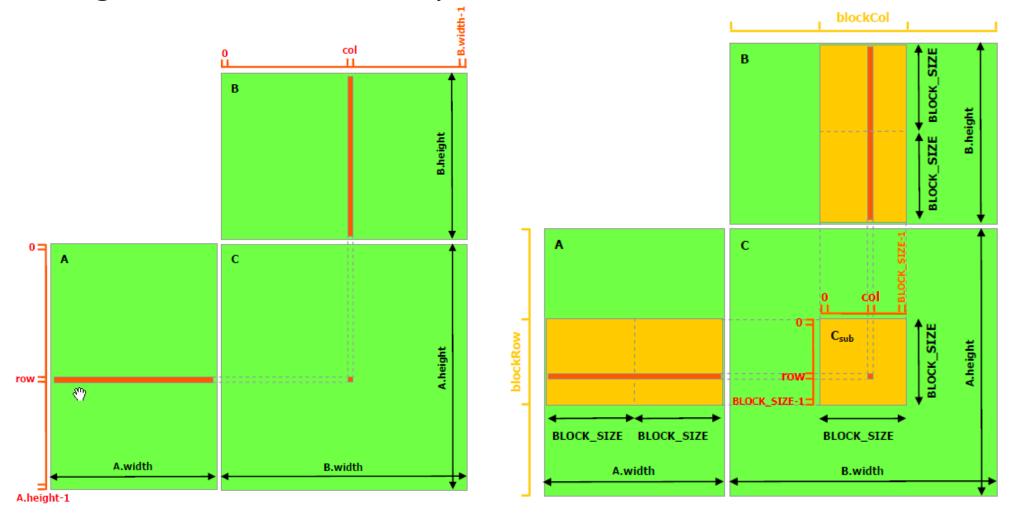
```
58 /** N =1024, 4 blocks, 256 threads/per block */
59 global void
60 matvec kernel shared(float * A, float * B, float * C, int N) {
       int i = blockDim.x * blockIdx.x + threadIdx.x; /* 0 - 1023 */
61
62
       int j;
63
64
       extern __shared__ float B_shared[]; /* the same size as B[1024] */
       B shared[i] = B[i];
65
       /* for block 0: 0-255 are filled */
66
67
       /* for block 1: 256-511 are filled */
68
       /* for block 2: 512-767 are filled */
69
       /* for block 3: 768 - 1023 are filled */
70
71
       B shared [(i+256) \times 1024] = B[(i+256) \times 1024];
72
       B shared [(i+512) \times 1024] = B[(i+512) \times 1024];
73
       B shared [(i+768) \times 1024] = B[(i+768) \times 1024];
74
75
       syncthreads();
76
       if (i < N) {
77
78
         float temp = 0.0;
79
         for (j=0; j<N; j++)</pre>
80
           temp += A[i*N+i] * B shared[i];
81
82
         C[i] = temp;
       }
83
84 }
```

### **Matrix Vector Multiplication**

```
global void
 86
 87
    matvec_kernel_shared_general(float * A, float * B, float * C, int N) {
 88
        int i = blockDim.x * blockIdx.x + threadIdx.x; /* 0 - 1023 */
 89
        int j;
 90
 91
        extern _____shared___ float B__shared[];
 92
        int k;
 93
        for (k=0; k<gridDim.x; k++) {</pre>
 94
            B_shared[(threadIdx.x + k*blockDim.x)%N] = B[(threadIdx.x + k*blockDim.x)%N];
 95
        }
 96
 97
        syncthreads();
 98
99
        if (i < N) {
          float temp = 0.0;
100
          for (j=0; j<N; j++)</pre>
101
102
            temp += A[i*N+j] * B shared[j];
103
104
          C[i] = temp;
105
        }
106 }
```

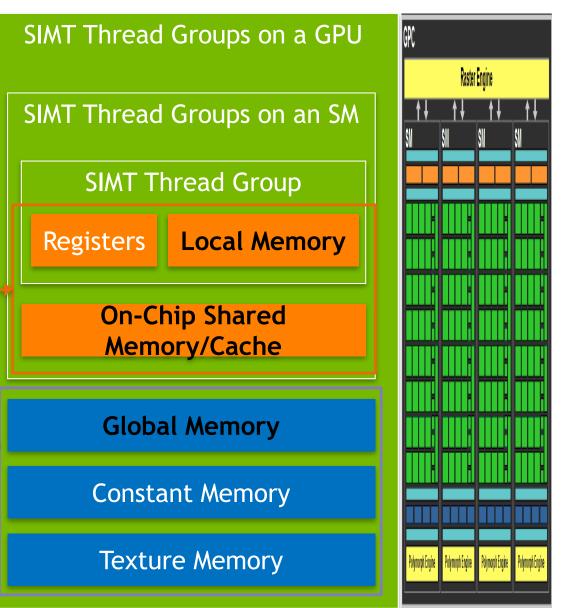
### Matrix Multiplication V1 and V2 in Assignment #4

 https://docs.nvidia.com/cuda/cuda-c-programmingguide/#shared-memory

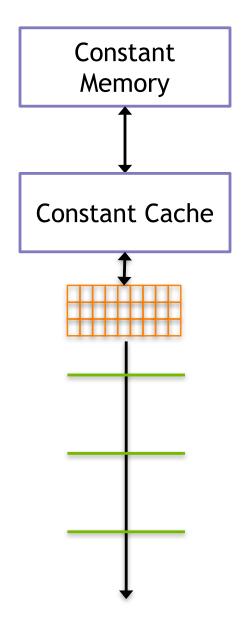


# **GPU Memory Hierarchy**

- More complex than the CPU memory
  - Many different types of memory, each with special-purpose characteristics
    - SRAM-
    - DRAM 、
  - More *explicit* control over data movement



- Declared with the \_\_\_\_constant\_ keyword
- Read-only
- Limited in size: 64KB
- Stored in device memory (same physical location as Global Memory)
- Cached in a per-SM constant cache
- Optimized for all threads in a warp accessing the same memory cell



- As its name suggests, constant memory is best used for storing constants
  - Values which are read-only
  - Values that are accessed identically by all threads
- For example: suppose all threads are evaluating the equation

y = mx + b

for different values of  ${\bf x}$  , but identical values of  ${\bf m}$  and  ${\bf b}$ 

- All threads would reference m and b with the same memory operation
- This broadcast access pattern is optimal for constant memory

- A simple 1D stencil
  - target cell is updated based on its 8 neighbors, weighted by some constants c0, c1, c2, c3



$$f'(x) \approx c_0 \left( f(x+4h) - f(x-4h) \right) + c_1 \left( f(x+3h) - f(x-3h) \right) \\ -c_2 \left( f(x+2h) - f(x-2h) \right) + c_3 \left( f(x+h) - f(x-h) \right)$$

 constantStencil.cu contains an example 1D stencil that uses constant memory

```
constant float coef[RADIUS + 1];
cudaMemcpyToSymbol(coef, h coef, (RADIUS + 1) *
sizeof(float));
 global void stencil_1d(float *in, float *out, int N)
  for (int i = 1; i <= RADIUS; i++) {</pre>
    tmp += coef[i] * (smem[sidx + i] - smem[sidx - i]);
  }
}
```

# **CUDA Synchronization**

- When using shared memory, you often must coordinate accesses by multiple threads to the same data
- CUDA offers synchronization primitives that allow you to synchronize among threads

# **CUDA Synchronization**

#### syncthreads

- Synchronizes execution across all threads in a thread block
- No thread in a thread block can progress past a \_\_\_\_syncthreads before all other threads have reached it
- <u>syncthreads</u> ensures that all changes to shared and global memory by threads in this block are visible to all other threads in this block

#### threadfence\_block

- All writes to shared and global memory by the calling thread are visible to all other threads in its block after this fence
- Does not block thread execution

# **CUDA Synchronization**

#### threadfence

- All writes to global memory by the calling thread are visible to all other threads in its grid after this fence
- Does not block thread execution

#### threadfence\_system

- All writes to global memory, page-locked host memory, and memory of other CUDA devices by the calling thread are visible to all other threads on all CUDA devices and all host threads after this fence
- Does not block thread execution

# **Suggested Readings**

- 1. Chapter 2, 4, 5 in *Professional CUDA C Programming*
- 2. Cliff Woolley. GPU Optimization Fundamentals. 2013. https://www.olcf.ornl.gov/wpcontent/uploads/2013/02/GPU\_Opt\_Fund-CW1.pdf
- 3. Mark Harris. Using Shared Memory in CUDA C/C++. http://devblogs.nvidia.com/ parallelforall/using-sharedmemory-cuda-cc/
- 4. Mark Harris. Optimizing Parallel Reduction in CUDA. http://developer.download.nvidia .com/assets/cuda/files/reduction.pdf