Lecture 16: Analytical Modeling of Parallel Programs: Metrics and Analysis

CSCE 569 Parallel Computing

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Topics

- Introduction
- Programming on shared memory system (Chapter 7)
  - **OpenMP**
- Principles of parallel algorithm design (Chapter 3)
- Programming on large scale systems (Chapter 6)
  - **MPI (point to point and collectives)**
  - Introduction to PGAS languages, UPC and Chapel

☛ **Analysis of parallel program executions (Chapter 5)**
  - **Performance Metrics for Parallel Systems**
    - **Execution Time, Overhead, Speedup, Efficiency, Cost**
  - **Scalability of Parallel Systems**
  - **Use of performance tools**

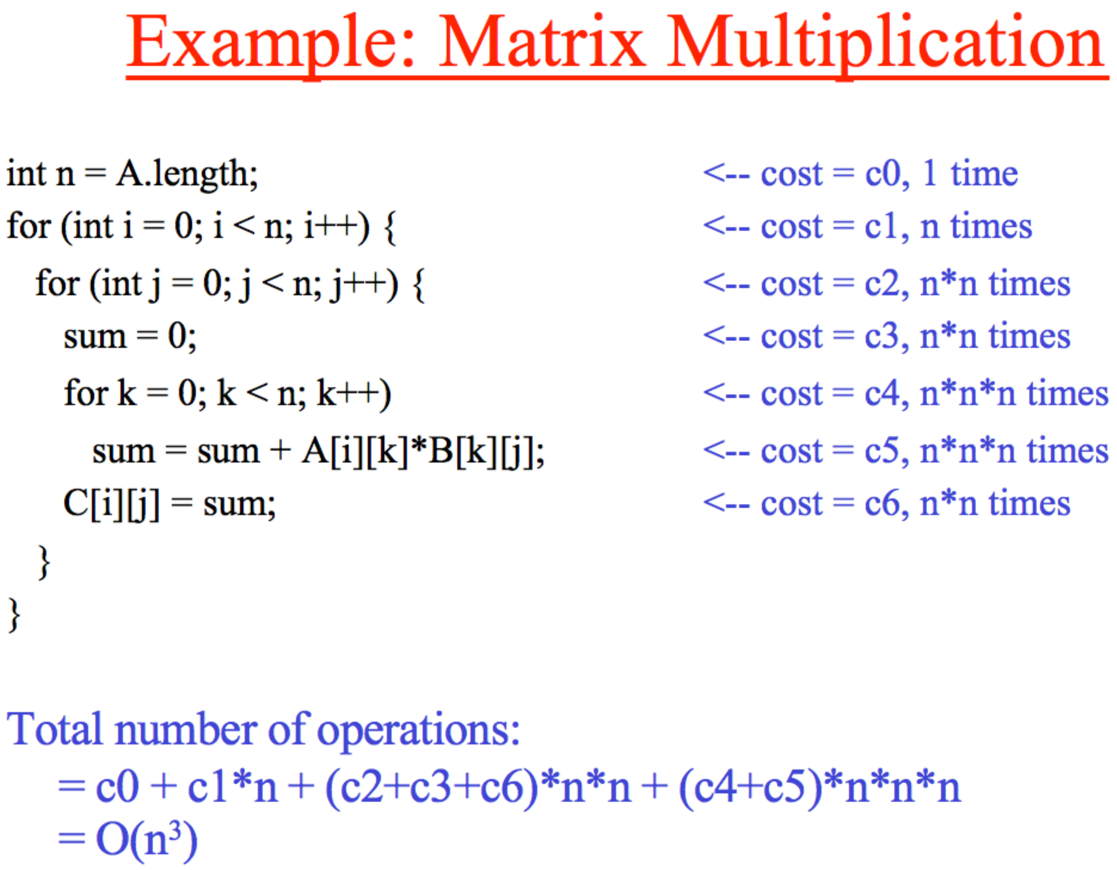
Topic Overview

☛ **Introduction**

- **Performance Metrics for Parallel Systems**
  - **Execution Time, Overhead, Speedup, Efficiency, Cost**
- **Amdahl’s Law**
- Scalability of Parallel Systems
  - Isoefficiency Metric of Scalability
- Minimum Execution Time and Minimum Cost-Optimal Execution Time
- Asymptotic Analysis of Parallel Programs
- Other Scalability Metrics
  - Scaled speedup, Serial fraction

Analytical Modeling: Sequential Execution Time

- The execution time of a sequential algorithm
  - Asymptotic execution time as **a function of input size**
    - **identical on any serial platform**



Example: Matrix Multiplication

```
int n = A.length;                          <-- cost = c0, 1 time
for (int i = 0; i < n; i++) {              <-- cost = c1, n times
  for (int j = 0; j < n; j++) {            <-- cost = c2, n*n times
    sum = 0;                               <-- cost = c3, n*n times
    for k = 0; k < n; k++)                 <-- cost = c4, n*n*n times
      sum = sum + A[i][k]*B[k][j];         <-- cost = c5, n*n*n times
    C[i][j] = sum;                         <-- cost = c6, n*n times
  }
}
```

Total number of operations:

$= c0 + c1*n + (c2+c3+c6)*n*n + (c4+c5)*n*n*n$

$= O(n^3)$

- Big-O Notation
  - $O(1)$
  - $O(N)$
  - $O(N^2)$
  - $O(N\log N)$
  - **$O(N^3)$**
  - …

Count the number of operations

Parallel Execution Time

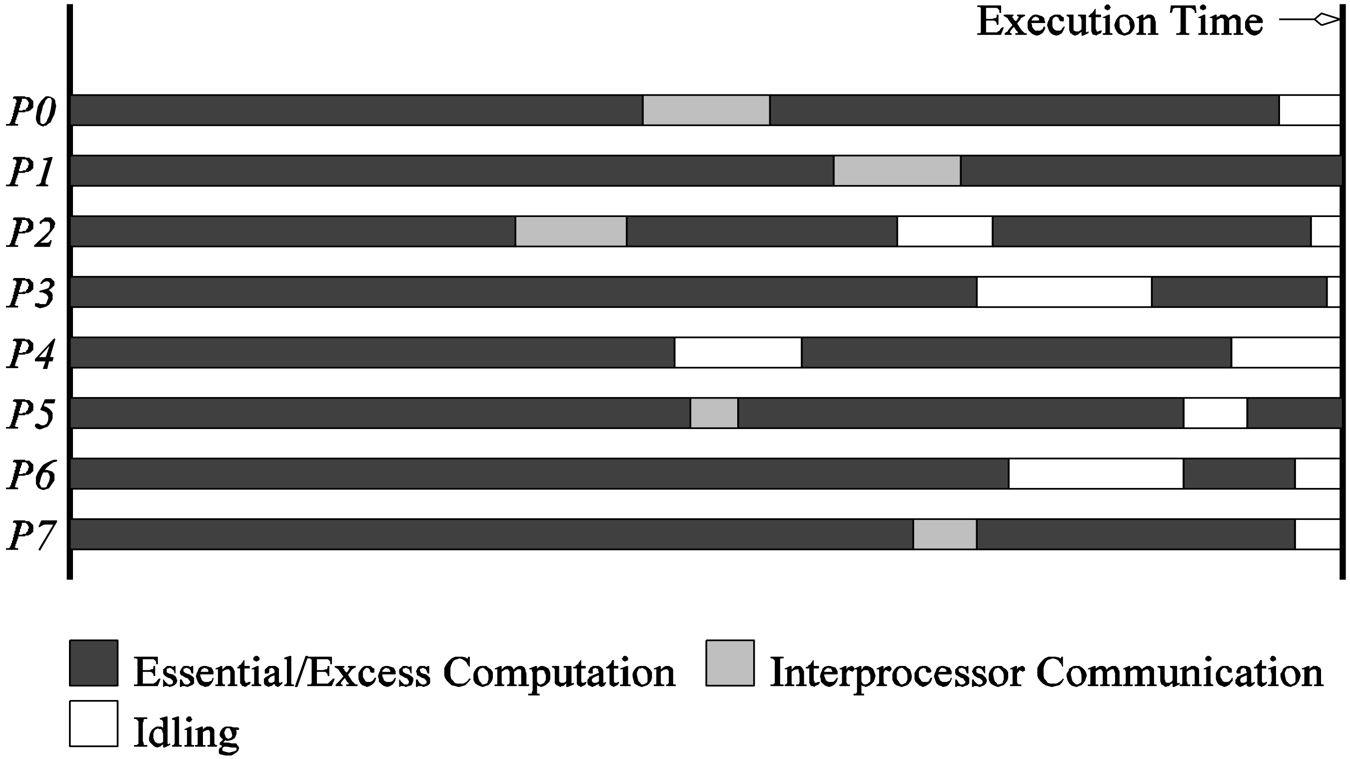
- Parallel execution time is a function of:
  - input size
  - **number of processors (machine performance)**
  - **communication parameters of target platform (network)**

- Implications
  - must analyze parallel program for a particular target platform
    - communication characteristics can differ by more than O(1)
  - **parallel program = parallel algorithm + platform**

Overhead in Parallel Programs

If using two processors, shouldn't a program run twice as fast?

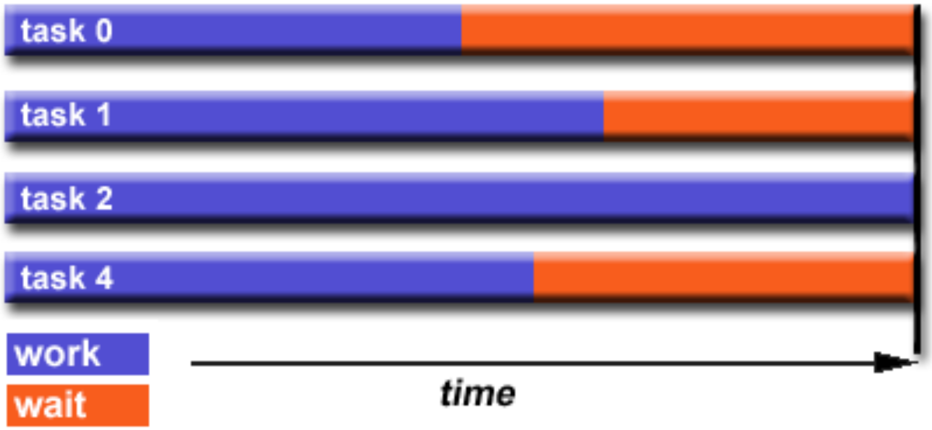
- Not all parts of the program are parallelized
- Certain amount of overheads incurred when doing it in parallel



Overheads in Parallel Programs

- Interprocess interactions:
  - Communication
    - Data movement
  - Synchronization/contention

- Idling:
  - Load imbalance
  - Synchronization
    - Sync itself has overhead
  - Serial components



- Excess computation
  - computation not performed by the serial version
    - E.g. replicated computation to minimize communication.

Topic Overview

- Introduction
- ☛ **Five Performance Metrics for Parallel Systems**
  - **Execution Time, Overhead, Speedup, Efficiency, Cost**
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- Scalability of Parallel Systems
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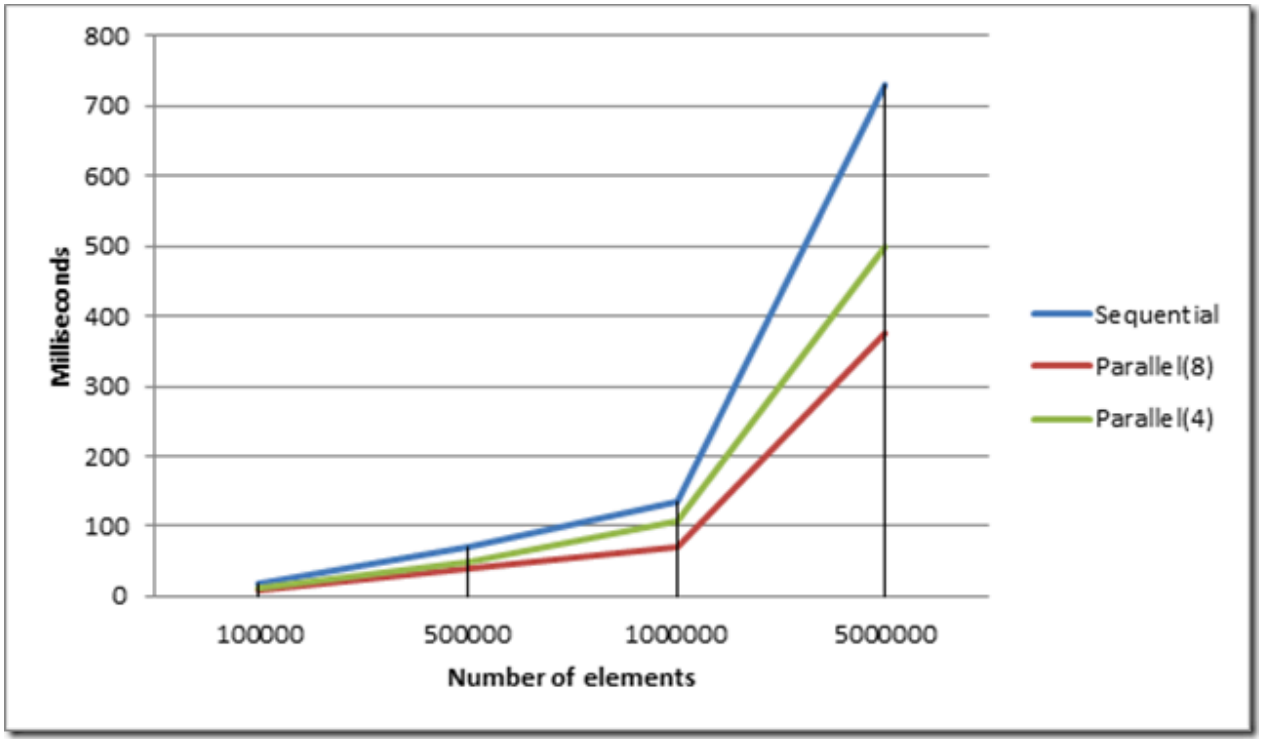
Performance Metrics #1: Execution Time

Does a parallel program run faster than its sequential version?

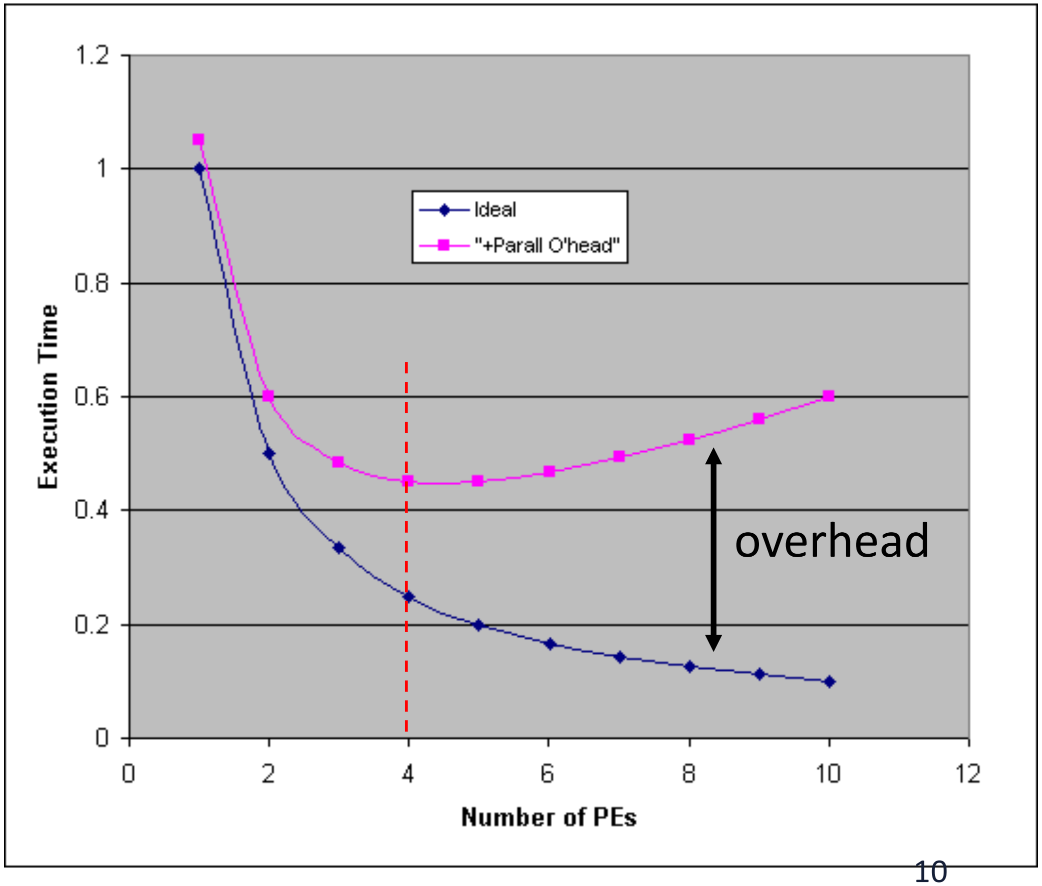
- Serial time: $\mathbf{T}_{s}$
  - time elapsed between the start and end of serial execution
- Parallel time: $\mathbf{T}_{\mathbf{p}}$
  - time elapsed between first process start and last process end



Performance Metrics #2: Parallel Overhead

What are the cost to enable parallelism?

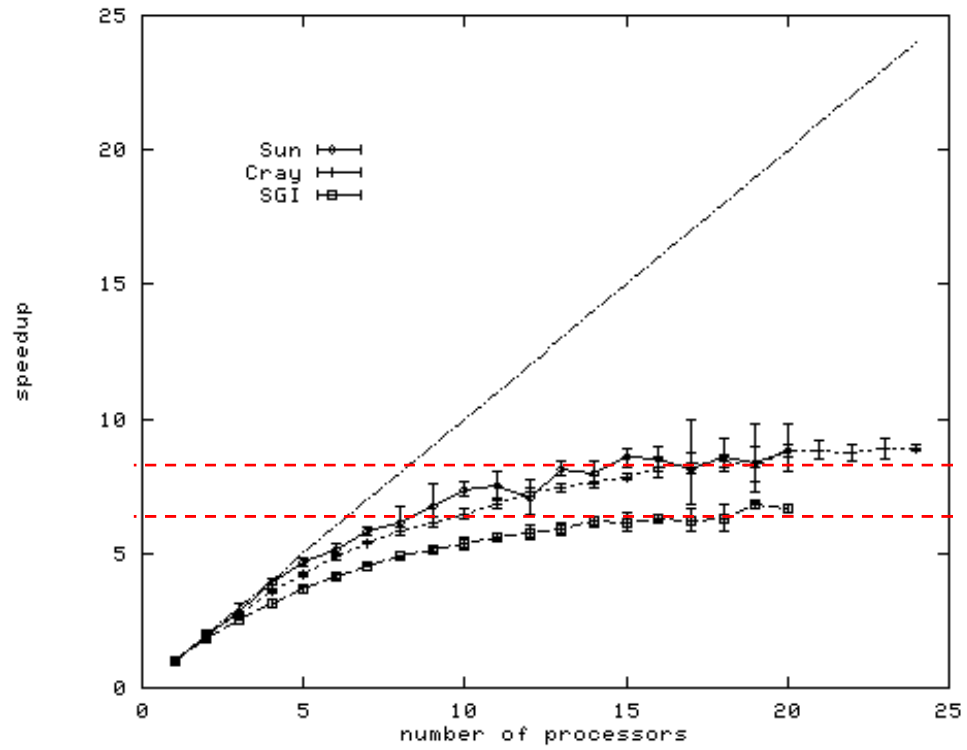
- T_{all} : the total time collectively spent by all the processors
  - $T_{all} = p\,T_P$ (p is the number of processors).
- T_S : serial execution time
- Total parallel overhead T_o
  - $T_o = T_{all} - T_S$
  - $T_o = p\,T_P - T_S$



Performance Metrics #3: Speedup

What is the benefit from increasing parallelism?

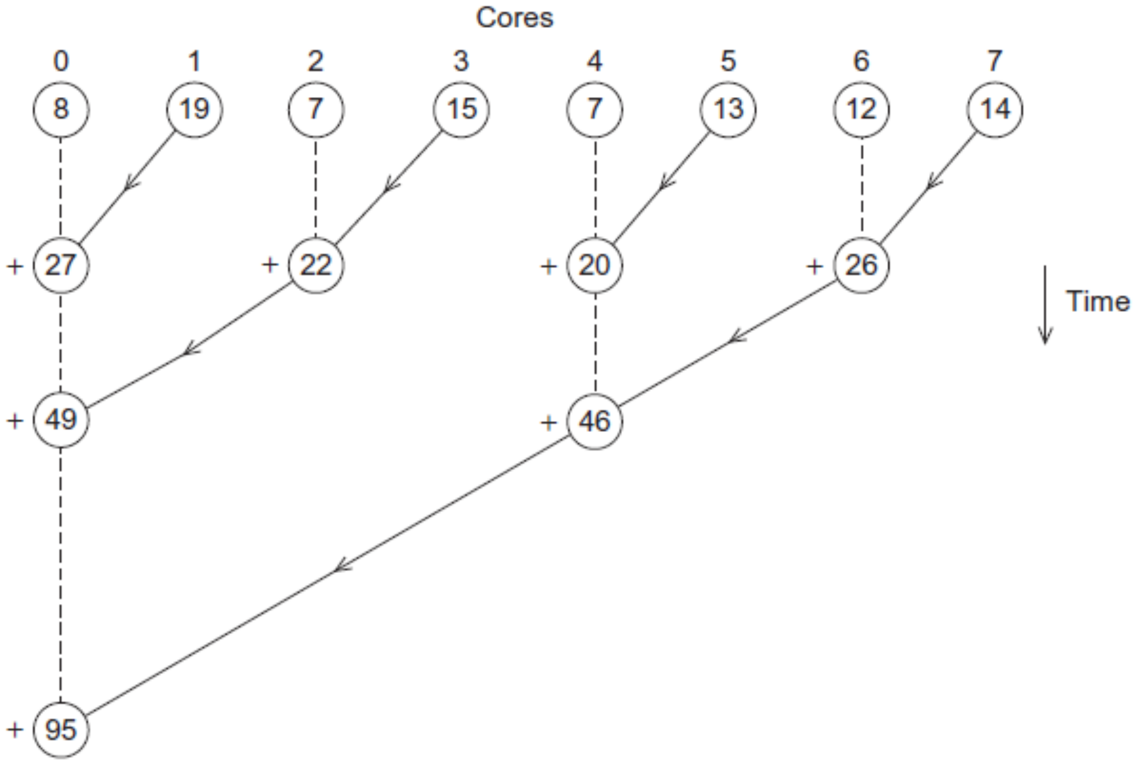
- Speedup (S): $\mathbf{T_S / T_P}$
  - The ratio of the time taken to solve a problem on a single processor to the time required to solve the same problem on a parallel computer with p identical processing elements.



Performance Metrics: Example

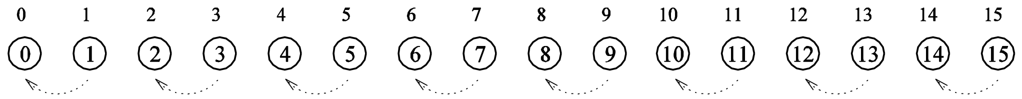
Adding *n* numbers

- Sequential: Θ (***n***)
- Using ***n*** processing elements.
  - If ***n*** is a power of two, in **log *n*** steps by propagating partial sums up a logical binary tree of processors.

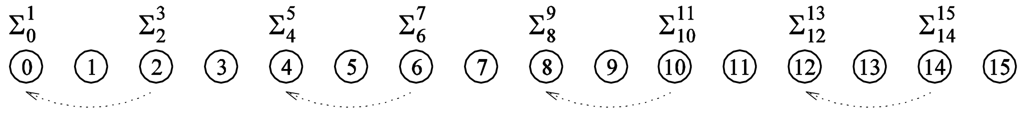


Performance Metrics: Example – cont'd

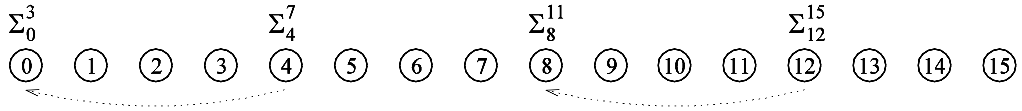
- **Σ^j_i** denotes the sum of numbers with consecutive labels from ***i*** to ***j***
- Analysis:
  - An addition takes t_c
  - Communication takes $t_s + t_w$
  - t_c and $(t_s + t_w)$ are constant
- Sequential and parallel time:
  - $T_S = \Theta(n)$
  - $T_P = \Theta(\log n)$
- Speedup ***S:***
  - $S = \Theta(n / \log n)$



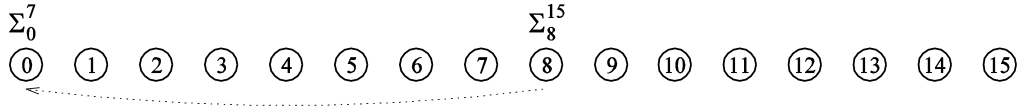
(a) Initial data distribution and the first communication step



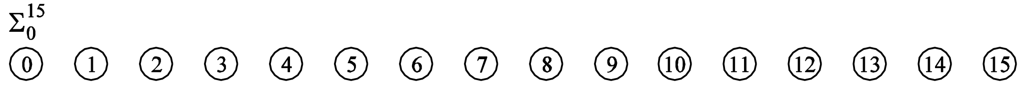
(b) Second communication step



(c) Third communication step



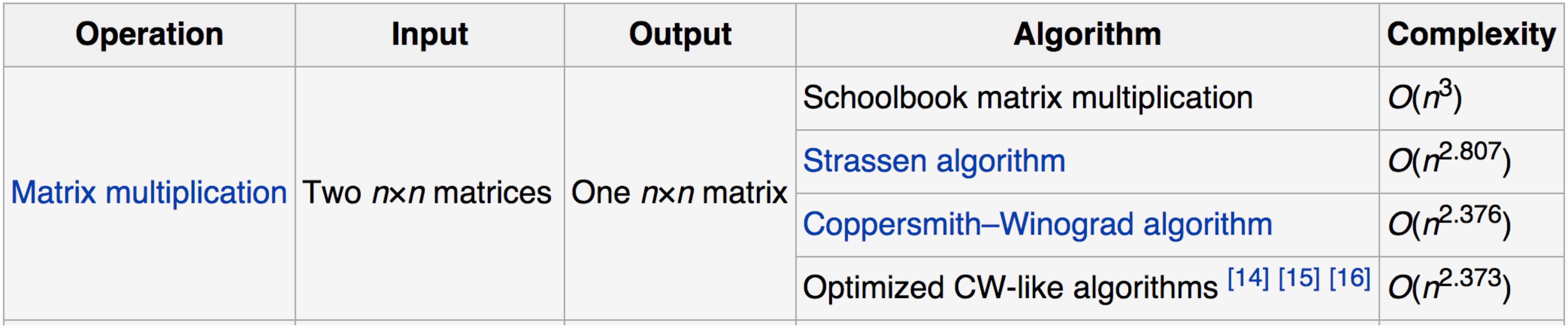
(d) Fourth communication step



(e) Accumulation of the sum at processing element 0 after the final communicatio

Performance Metrics #3: Speedup

- **The yardstick: T_s**
  - Many serial algorithms available, each with different asymptotic execution time
  - The parallelization of those algorithms varies too



Operation	Input	Output	Algorithm	Complexity
Matrix multiplication	Two $n \times n$ matrices	One $n \times n$ matrix	Schoolbook matrix multiplication	$O(n^3)$
			Strassen algorithm	$O(n^{2.807})$
			Coppersmith–Winograd algorithm	$O(n^{2.376})$
			Optimized CW-like algorithms [14] [15] [16]	$O(n^{2.373})$

http://en.wikipedia.org/wiki/Computational_complexity_of_mathematical_operations

Speedup Example: Sorting

```
procedure bubbleSort( A : vector)
  n := length( A )
  do
    swapped := false
    n := n - 1
    for each i in 0 to n - 1
      if A[i] > A[i + 1]
        swap(A[i], A[i + 1]); swapped := true
  while (swapped)
end procedure
```

Odd-even sort

"parallel bubble sort"

```
procedure oddEvenSort( A : vector)
  n := length( A )
  do
    swapped := false
    for each i in 0 to n - 1 by 2 in parallel
      if A[i] > A[i + 1]
        swap(A[i], A[i + 1]); swapped := true
    for each i in 1 to n - 1 by 2 in parallel
      if A[i] > A[i + 1]
        swap(A[i], A[i + 1]); swapped := true
  while (swapped)
end procedure
```

http://en.wikipedia.org/wiki/Sorting_algorithm

Speedup Example: Sorting – cont’d

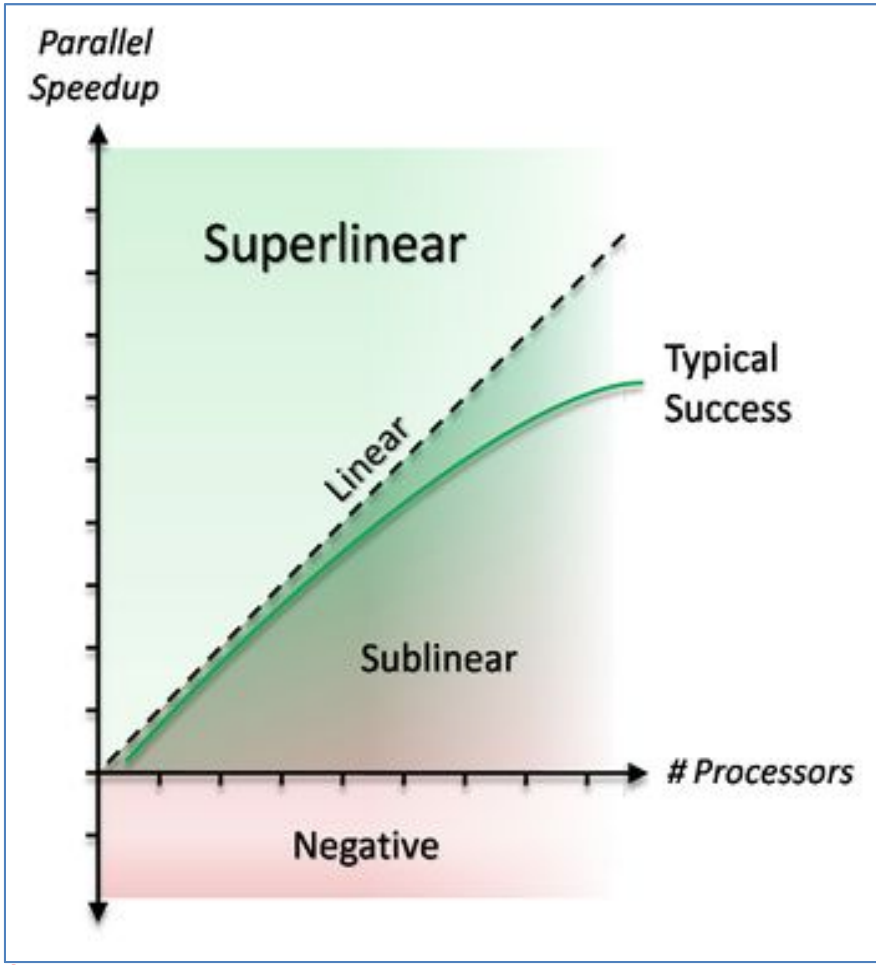
- The serial execution time for bubblesort:150 seconds.
- Odd-even parallel bubble sort: is 40 seconds.
- The speedup: 150/40 = 3.75.
  - But is this really a fair assessment of the system?

- What if serial quicksort only took 30 seconds?
- In this case, the speedup is 30/40 = 0.75
  - A more realistic assessment

- **In reality, consider the best sequential program as baseline**
  - **Not even the parallel program running with 1 PE**
    - **We do this in our assignment**

Performance Metrics: Speedup Bounds

- Speedup, in theory, should be upper bounded by ***p***
  - We can only expect a ***p***-fold speedup if we use ***p*** times as many resources.

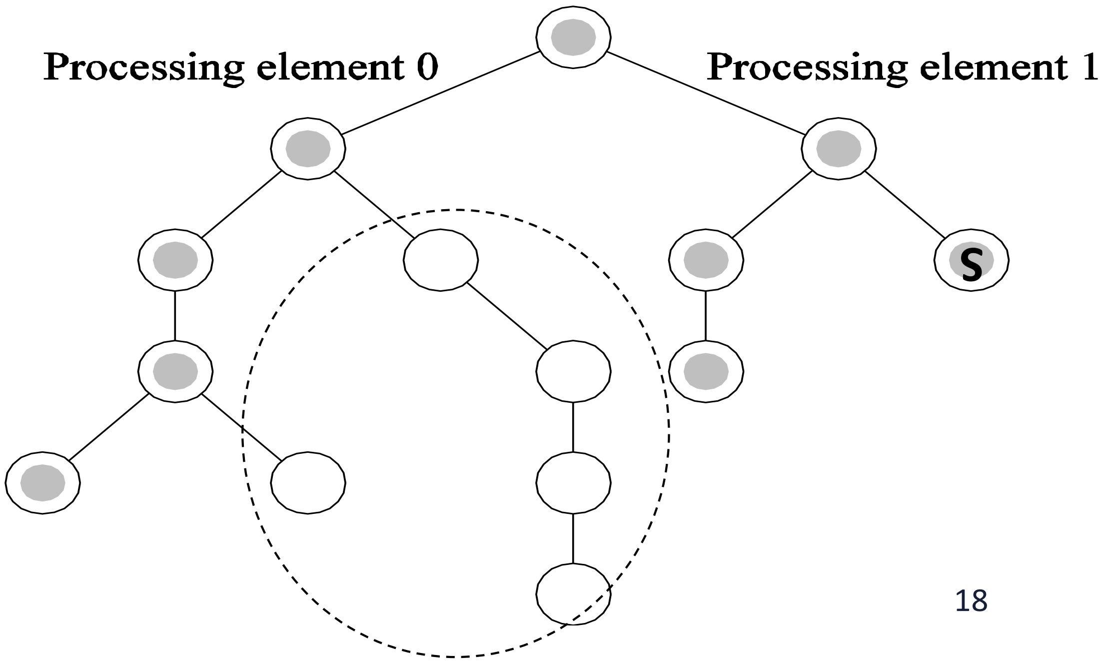


- ***Theoretically***, a speedup greater than ***p*** is possible only if each processor spends less than T_S / p time solving the problem.
  - Violate the rules of using the best sequential as baseline
- Speedups:
  - Linear
  - Sublinear
  - Superlinear
- ***In practice, superlinear is possible***

Performance Metrics: Superlinear Speedups

Parallel algorithm does less work than its serial versions

- Searching node '**S**' in an unstructured tree
- Parallel with two PEs using **depth-first traversal**
  - PE 0 searching the left subtree expands only the shaded nodes before the solution is found by PE 1
  - PE 1 searching the right subtree
- Serial algorithm expands the entire tree
  - Does more work than the parallel algorithm.



Performance Metrics: Superlinear Speedups

Resource-based superlinearity

- Parallel execution:
  - The higher aggregate cache/memory bandwidth can result in better cache-hit ratios, and therefore superlinearity.

- Example: A processor with 64KB of cache yields an 80% hit ratio. If two processors are used, since the problem size/processor is smaller, the hit ratio goes up to 90%. Of the remaining 10% access, 8% come from local memory and 2% from remote memory.
- If DRAM access time is 100 ns, cache access time is 2 ns, and remote memory access time is 400ns, this corresponds to a speedup of 2.43!

Performance Metrics #4: Efficiency

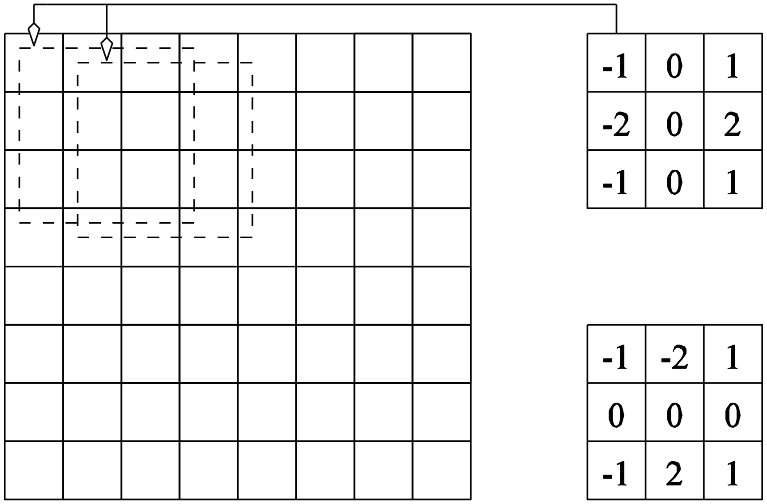
- Fraction of time for which a process perform useful work

$$E = S / p = T_S / (p\ T_P)$$

- Bounds
  - Theoretically, **$0 \le E \le 1$**
    - **The larger, the better**
    - **E=1: 0 overhead**
  - Practically, **E > 1 if superlinear speedup is achieved**
- Previous example: adding N numbers using N PEs
  - **Speedup:** $S = \Theta(N / \log N)$
  - **Efficiency:** $E = S/N = \Theta(N / \log N) / N = \Theta(1 / \log N)$
    - **Very low when N is big**

Example: Image Filtering (e.g. Edge Detection)

- Apply 3x3 template to each pixel of the images
  - Stencil computation
- **Serial performance: $T_S = 9t_c\, n^2$**
  - Each pixel has 9 multiply-add (MA)
    - Each MA takes constant t_c time
  - An n x n image for n^2 pixels



-1	0	1
-2	0	2
-1	0	1

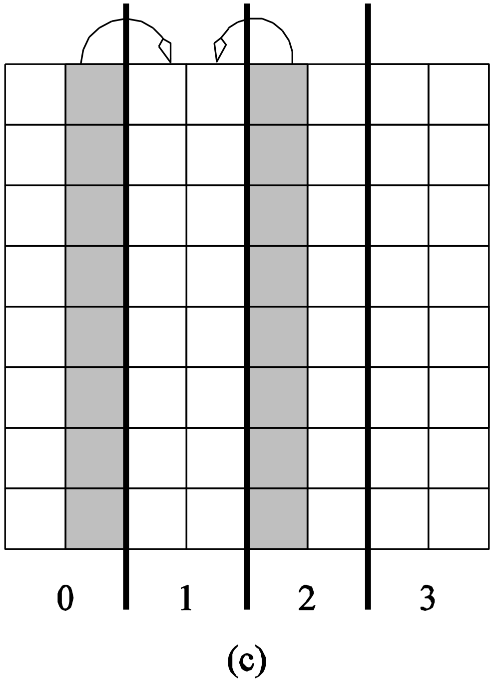
-1	-2	1
0	0	0
-1	2	1

http://en.wikipedia.org/wiki/Edge_detection

(a) (b)

Edge Detection: Parallel Version

- Partitions the image equally into vertical segments, each with n^2 / p pixels.



- Computation by each PE: $T_S = 9\,t_c n^2 / p$
- Communications by each PE: $2(t_s + t_w n)$
  - The boundary of each segment is **2*n*** pixels
    - Two boundaries: left and right
  - Each boundary exchange takes $t_s + t_w n$
- **Parallel performance:** $T_P = 9t_c\dfrac{n^2}{p} + 2(t_s + t_w n)$

Edge Detection: Parallel Speedup and Efficiency

- Serial performance: $T_S = 9t_c n^2$
- Parallel performance: $$T_P = 9t_c \frac{n^2}{p} + 2(t_s + t_w n)$$
- **Speedup: $S = T_s/T_p$**

$$S = \frac{9t_c n^2}{9t_c \frac{n^2}{p} + 2(t_s + t_w n)}$$

- **Efficiency: $E = S/p$**

$$E = \frac{1}{1 + \frac{2p(t_s + t_w n)}{9t_c n^2}}.$$

Performance Metrics #5: Cost

Product of parallel execution time and number of PEs: $p*T_P$

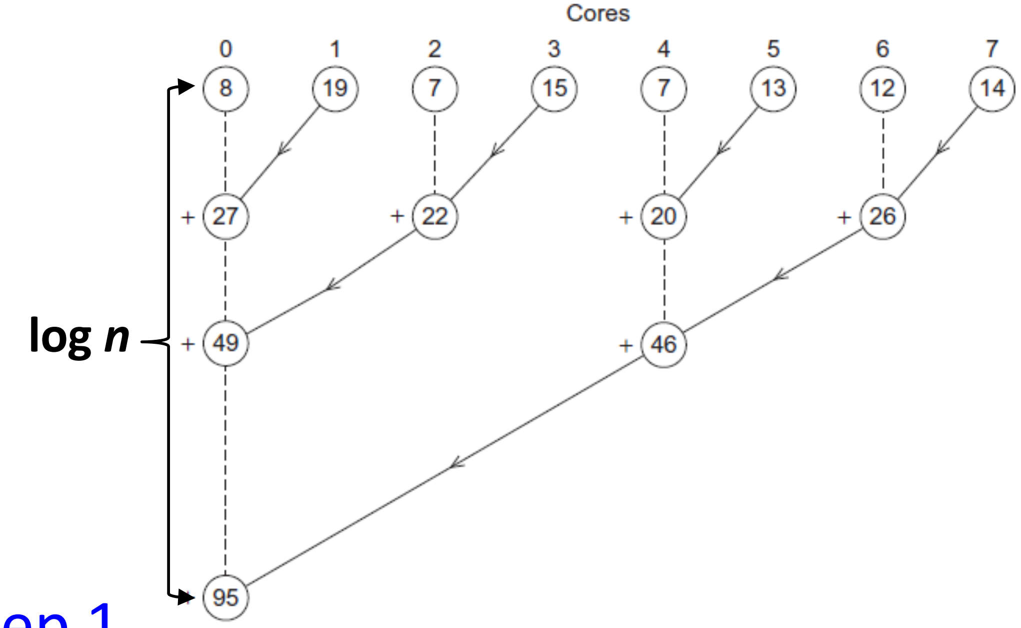
- The total amount of time by all PEs to solve the problem

- *Cost-optimal* : parallel cost ≅ serial cost
  - ~0 overhead
  - **E = Θ (1), since E = $T_S / p*T_P$**

Cost: An Example

Adding n numbers on n PEs

- Serial performance: $T_S = \Theta(n)$
- Parallel performance: $T_P = \Theta(\log n)$
- Cost: $p\,T_P = \Theta(n \log n)$
- Optimal or not:
  - $E = n/n * \log n = \Theta(1/\log n)$
  - **Not cost-optimal.**



- **Why not optimal**
  - Waste of CPU cycles after step 1
    - Only core 0 is doing all the useful work in logN times

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Amdahl’s Law

Amdahl’s law for overall speedup

$$\text{Overall Speedup} = \frac{1}{(1-F)+\frac{F}{S}}$$

F = The fraction enhanced

S = The speedup of the enhanced fraction

- The word “law” is often used by computer scientists when it is an observed phenomena (e.g, Moore’s Law) and not a theorem that has been proven in a strict sense.

Gene Amdahl, "Validity of the single processor approach to achieving large-scale computing capabilities", AFIPS Conference Proceedings, 30:483-485, **1967**.

Using Amdahl's Law

Overall speedup if we make 90% of a program run 10 times faster.

$F = 0.9 \quad S = 10$

$$\text{Overall Speedup} = \frac{1}{(1-0.9)+\frac{0.9}{10}} = \frac{1}{0.1+0.09} = 5.26$$

Overall speedup if we make 80% of a program run 20% faster.

$F = 0.8 \quad S = 1.2$

$$\text{Overall Speedup} = \frac{1}{(1-0.8)+\frac{0.8}{1.2}} = \frac{1}{0.2+0.66} = 1.153$$

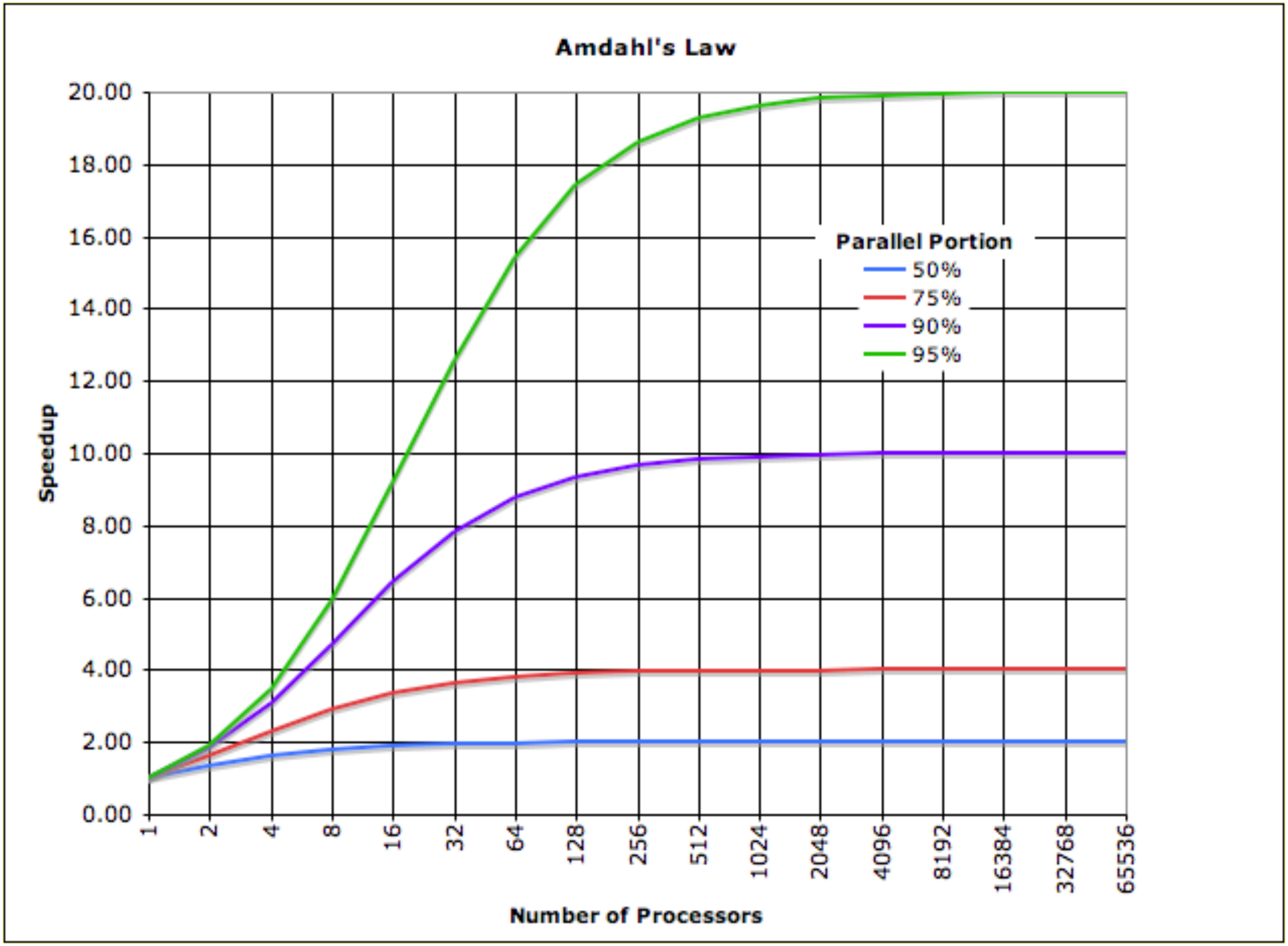
Amdahl’s Law for Parallelism

- The enhanced fraction F is through parallelism, perfect parallelism with linear speedup
  - The speedup for F is N for N processors
- Overall speedup

$$S(N) = \frac{T_s}{T_p} = \frac{T_s}{(1-F) * T_s + \frac{F * T_s}{N}} = \frac{1}{1-F+\frac{F}{N}}$$

- Speedup upper bound (when N →∞): $S(N) \le \frac{1}{1-F}$
  - 1-F: the sequential portion of a program

Amdahl's Law for Parallelism



Amdahl’s Law Usefulness

- Amdahl’s law is valid for traditional problems and has several useful interpretations.
- Some textbooks show how Amdahl’s law can be used to increase the efficient of parallel algorithms
  - $E=(1/((1-F)+F/N))/N = 1/(N(1-F)+F)$
    - If we increase N, and the problem size in certain rate(so F increased), we can still keep E constant
- Amdahl’s law shows that efforts required to further reduce the fraction of the code that is sequential may pay off in large performance gains.
- Hardware that achieves even a small decrease in the percent of things executed sequentially may be considerably more efficient.

Amdahl’s Law for Parallelism

- However: for long time, Amdahl’s law was viewed as a fatal flaw to the usefulness of parallelism
  - Focuses a particular algorithm and problem sizes, and does not consider that other algorithms with more parallelism may exist, or scalability issues
  - Amdahl’s law applies only to “standard” problems were superlinearity can not occur
  - **Gustafon’s Law:** The proportion of the computations that are sequential normally decreases as the problem size increases.
- Currently, it is generally accepted by parallel computing professionals that Amdahl’s law is not a serious limit the benefit and future of parallel computing.

Compilers and More: Is Amdahl’s Law Still Relevant? Michael Wolfe, http://www.hpcwire.com/2015/01/22/compilers-amdahls-law-still-relevant/, 01/22/2015

References

- Adapted from slides “Principles of Parallel Algorithm Design” by Ananth Grama
- “Analytical Modeling of Parallel Systems”, Chapter 5 in Ananth Grama, Anshul Gupta, George Karypis, and Vipin Kumar, Introduction to Parallel Computing'', “ Addison Wesley, 2003.