Lecture 10: Principles of Parallel Algorithm Design

CSCE 569 Parallel Computing

Department of Computer Science and Engineering Yonghong Yan yanyh@cse.sc.edu http://cse.sc.edu/~yanyh

Last two lectures: Algorithms and Concurrency

- Introduction to Parallel Algorithms
 - Tasks and decomposition
 - Processes and mapping
- Decomposition Techniques
 - Recursive decomposition (divide-conquer)
 - Data decomposition (input, output, input+output, intermediate)
- Terms and concepts
 - Task dependency graph, task granularity, degree of concurrency
 - Task interaction graph, critical path
- Examples:
 - Dense vector addition, matrix vector and matrix matrix product
 - Database query, quicksort, MIN
 - Image convolution(filtering) and Jacobi

Today's lecture

Decomposition Techniques - continued

- Exploratory Decomposition
- Hybrid Decomposition

Mapping tasks to processes/cores/CPU/PEs

- Characteristics of Tasks and Interactions
 - Task Generation, Granularity, and Context
 - Characteristics of Task Interactions
- Mapping Techniques for Load Balancing
 - Static and Dynamic Mapping
- Methods for Minimizing Interaction Overheads
- Parallel Algorithm Design Models

Exploratory Decomposition

- Decomposition is fixed/static from the design
 - Data and recursive

- Exploration (search) of a state space of solutions
 - Problem decomposition reflects shape of execution
 - Goes hand-in-hand with its execution
- Examples
 - discrete optimization, e.g. 0/1 integer programming
 - theorem proving
 - game playing

Exploratory Decomposition: Example

Solve a 15 puzzle

• Sequence of three moves from state (a) to final state (d)



• From an arbitrary state, must search for a solution

Exploratory Decomposition: Example

Solving a 15 puzzle

- Search
 - generate successor states of the current state
 - explore each as an independent task



final state (solution)

Exploratory Decomposition Speedup

Solve a 15 puzzle

- The decomposition behaves according to the parallel formulation
 - May change the amount of work done



Execution terminate when a solution is found

Speculative Decomposition

- Dependencies between tasks are not known a-priori.
 - Impossible to identify independent tasks
- Two approaches
 - Conservative approaches, which identify independent tasks only when they are guaranteed to not have dependencies
 - May yield little concurrency
 - Optimistic approaches, which schedule tasks even when they may potentially be inter-dependent
 - Roll-back changes in case of an error

Speculative Decomposition: Example

Discrete event simulation

- Centralized time-ordered event list
 - you get up →get ready→drive to work→work→eat lunch→
 work some more→drive back→eat dinner→and sleep
- Simulation
 - extract next event in time order
 - process the event
 - if required, insert new events into the event list
- Optimistic event scheduling
 - assume outcomes of all prior events
 - speculatively process next event
 - if assumption is incorrect, roll back its effects and continue

Speculative Decomposition: Example

Simulation of a network of nodes

- Simulate network behavior for various input and node delays
 - The input are dynamically changing
 - Thus task dependency is unknown



System Components

- Speculate execution: tasks' input
 - Correct: parallelism
 - Incorrect: rollback and redo

Speculative vs Exploratory

- Exploratory decomposition
 - The output of multiple tasks from a branch is unknown
 - Parallel program perform more, less or same amount of work as serial program
- Speculative
 - The input at a branch leading to multiple parallel tasks is unknown
 - Parallel program perform more or same amount of work as the serial algorithm

Hybrid Decompositions

Use multiple decomposition techniques together

- One decomposition may be not optimal for concurrency
 - Quicksort recursive decomposition limits concurrency (Why?)



Combined recursive and data decomposition for MIN



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Characteristics of Tasks

- Theory
 - Decomposition: to parallelize theoretically
 - Concurrency available in a problem
- Practice
 - Task creations, interactions and mapping to PEs.
 - Realizing concurrency practically
 - Characteristics of tasks and task interactions
 - Impact choice and performance of parallelism
- Characteristics of tasks
 - Task generation strategies
 - Task sizes (the amount of work, e.g. FLOPs)
 - Size of data associated with tasks

Task Generation

- Static task generation
 - Concurrent tasks and task graph known a-priori (before execution)
 - Typically using recursive or data decomposition
 - Examples
 - Matrix operations
 - Graph algorithms
 - Image processing applications
 - Other *regularly* structured problems
- Dynamic task generation
 - Computations formulate concurrent tasks and task graph on the fly
 - Not explicit a priori, though high-level rules or guidelines known
 - Typically by exploratory or speculative decompositions.
 - Also possible by recursive decomposition, e.g. quicksort
 - A classic example: game playing
 - 15 puzzle board

Task Sizes/Granularity

- The amount of work \rightarrow amount of time to complete
 - E.g. FLOPs, #memory access
- Uniform:
 - Often by even data decomposition, i.e. regular
- Non-uniform
 - Quicksort, the choice of pivot



Size of Data Associated with Tasks

- May be small or large compared to the task sizes
 - How relevant to the input and/or output data sizes
 - Example:
 - size(input) < size(computation), e.g., 15 puzzle
 - size(input) = size(computation) > size(output), e.g., min
 - size(input) = size(output) < size(computation), e.g., sort
- Considering the efforts to reconstruct the same task context
 - small data: small efforts: task can easily migrate to another process
 - large data: large efforts: ties the task to a process
- Context reconstructing vs communicating
 - It depends

- Aspects of interactions
 - What: shared data or synchronizations, and sizes of the media
 - When: the timing
 - Who: with which task(s), and overall topology/patterns
 - Do we know details of the above three before execution
 - How: involve one or both?
 - The implementation concern, implicit or explicit

Orthogonal classification

- Static vs. dynamic
- Regular vs. irregular
- Read-only vs. read-write
- One-sided vs. two-sided

- Aspects of interactions
 - What: shared data or synchronizations, and sizes of the media
 - When: the timing
 - Who: with which task(s), and overall topology/patterns
 - Do we know details of the above three before execution
 - How: involve one or both?
- Static interactions
 - Partners and timing (and else) are known a-priori
 - Relatively simpler to code into programs.
- Dynamic interactions
 - The timing or interacting tasks cannot be determined a-priori.
 - Harder to code, especially using explicit interaction.

- Aspects of interactions
 - What: shared data or synchronizations, and sizes of the media
 - When: the timing
 - Who: with which task(s), and overall topology/patterns
 - Do we know details of the above three before execution
 - How: involve one or both?
- Regular interactions
 - Definite pattern of the interactions
 - E.g. a mesh or ring
 - Can be exploited for efficient implementation.
- Irregular interactions
 - lack well-defined topologies
 - Modeled as a graph

Example of *Regular* Static Interaction

Image processing algorithms: dithering, edge detection

Nearest neighbor interactions on a 2D mesh



Example of Irregular Static Interaction

Sparse matrix vector multiplication



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(a)

(b)

- Aspects of interactions
 - What: shared data or synchronizations, and sizes of the media



- Read-only interactions
 - Tasks only read data items associated with other tasks
- Read-write interactions
 - Read, as well as modify data items associated with other tasks.
 - Harder to code
 - Require additional synchronization primitives
 - to avoid read-write and write-write ordering races

- Aspects of interactions
 - What: shared data or synchronizations, and sizes of the media
 - When: the timing
 - Who: with which task(s), and overall topology/patterns
 - Do we know details of the above three before execution
 - How: involve one or both?
 - The implementation concern, implicit or explicit
- One-sided
 - initiated & completed independently by 1 of 2 interacting tasks
 - GET and PUT
- Two-sided
 - both tasks coordinate in an interaction
 - SEND + RECV



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Today's lecture

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Mapping tasks to processes/cores/CPU/PEs

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Mapping Techniques

- Parallel algorithm design
 - Program decomposed
 - Characteristics of task and interactions identified

Assign large amount of concurrent tasks to equal or relatively small amount of processes for execution

• Though often we do 1:1 mapping

Mapping Techniques

- Goal of mapping: minimize overheads
 - There is cost to do parallelism
 - Interactions and idling(serialization)
- Contradicting objectives: interactions vs idling
 - Idling (serialization) \hat{U} : insufficient parallelism
 - Interactions \hat{U} : excessive concurrency
 - E.g. Assigning all work to one processor trivially minimizes interaction at the expense of significant idling.

Mapping Techniques for Minimum Idling

- Execution: alternating stages of computation and interaction
- Mapping must simultaneously minimize idling and load balance
 - Idling means not doing useful work
 - Load balance: doing the same amount of work
- Merely balancing load does not minimize idling



Mapping Techniques for Minimum Idling

Static or dynamic mapping

- Static Mapping
 - Tasks are mapped to processes a-prior
 - Need a good estimate of task sizes
 - Optimal mapping may be NP complete
- Dynamic Mapping
 - Tasks are mapped to processes at runtime
 - Because:
 - Tasks are generated at runtime
 - Their sizes are not known.
- Other factors determining the choice of mapping techniques
 - the size of data associated with a task
 - the characteristics of inter-task interactions
 - even the programming models and target architectures

Schemes for Static Mapping

- Mappings based on data decomposition
 - Mostly 1-1 mapping
- Mappings based on task graph partitioning
- Hybrid mappings

Mappings Based on Data Partitioning

- Partition the computation using a combination of
 - Data decomposition
 - The ``owner-computes'' rule

Example: 1-D block distribution of 2-D dense matrix

1-1 mapping of task/data and process

P_0
P_1
P_2
P_3
P_4
P_5
P_6
P_7

row-wise distribution

column-wise distribution



Block Array Distribution Schemes

Multi-dimensional Block distribution

P ₀	<i>P</i> ₁	<i>P</i> ₂	<i>P</i> ₃
P_4	P ₅	P_6	<i>P</i> ₇
P_8	P_9	P_{10}	<i>P</i> ₁₁
P ₁₂	P ₁₃	<i>P</i> ₁₄	P ₁₅









In general, higher dimension decomposition allows the use of larger # of processes.

Multiplying two dense matrices: A * B = C

- Partition the output matrix **C** using a block decomposition
 - Load balance: Each task compute the same number of elements of *C*
 - Note: each element of *C* corresponds to a single dot product
 - The choice of precise decomposition: 1-D (row/col) or 2-D
 - Determined by the associated communication overhead

ł	-		-, r	-		•	
	A(11)	A(12) A	(13)	B(11) B(12)	B(13)	C(11) C(12)	C(13)
	A(21)	A(22) A	(23) *	B(21) B(22)	B(23) = 0	2(21) C(22)	C(23)
	A(31)	A(32) A	.(33)	B(31) B(32)	B(33)	(31) C(32)	C(33)
	C(11) C(21) C(31)	= A(ll) = A(2l) = A(3l)	*B(11) + *B(11) + *B(11) +	A(12)*B(21) A(22)*B(21) A(32)*B(21)	+ A(13)*B(3 + A(23)*B(3 + A(33)*B(3	1) 1) 31)	
	C(12) C(22) C(32)	= A(11) = A(21) = A(31)	*B(12) + *B(12) + *B(12) +	A(12)*B(22) A(22)*B(22) A(32)*B(22)	+ A(13)*B(3 + A(23)*B(3 + A(33)*B(3	12) 12) 12)	
	C(13) C(23) C(33)	= A(11) = A(21) = A(31)	*B(13) + *B(13) + *B(13) +	A(12)*B(23) A(22)*B(23) A(32)*B(23)	+ A(13)*B(3 + A(23)*B(3 + A(33)*B(3	3) 3) 3)	

Block Distribution and Data Sharing for Dense Matrix Multiplication



 P_3

 P_{7}

 P_{11}

 P_{15}

Cyclic and Block Cyclic Distributions

- Consider a block distribution for LU decomposition (Gaussian Elimination)
 - The amount of computation per data item varies
 - Block decomposition would lead to significant load imbalance





After this iteration, column A[k + 1 : n, k] is logically the kth column of L and row A[k, k : n] is logically the kth row of U.

LU Factorization of a Dense Matrix

A decomposition of LU factorization into 14 tasks

$$\begin{pmatrix} A_{1,1} & A_{1,2} & A_{1,3} \\ A_{2,1} & A_{2,2} & A_{2,3} \\ A_{3,1} & A_{3,2} & A_{3,3} \end{pmatrix} \to \begin{pmatrix} L_{1,1} & 0 & 0 \\ L_{2,1} & L_{2,2} & 0 \\ L_{3,1} & L_{3,2} & L_{3,3} \end{pmatrix} . \begin{pmatrix} U_{1,1} & U_{1,2} & U_{1,3} \\ 0 & U_{2,2} & U_{2,3} \\ 0 & 0 & U_{3,3} \end{pmatrix}$$

1:
$$A_{1,1} \rightarrow L_{1,1}U_{1,1}$$
6: $A_{2,2} = A_{2,2} - L_{2,1}U_{1,2}$ 11: $L_{3,2} = A_{3,2}U_{2,2}^{-1}$ 2: $L_{2,1} = A_{2,1}U_{1,1}^{-1}$ 7: $A_{3,2} = A_{3,2} - L_{3,1}U_{1,2}$ 12: $U_{2,3} = L_{2,2}^{-1}A_{2,3}$ 3: $L_{3,1} = A_{3,1}U_{1,1}^{-1}$ 8: $A_{2,3} = A_{2,3} - L_{2,1}U_{1,3}$ 13: $A_{3,3} = A_{3,3} - L_{3,2}U_{2,3}$ 4: $U_{1,2} = L_{1,1}^{-1}A_{1,2}$ 9: $A_{3,3} = A_{3,3} - L_{3,1}U_{1,3}$ 14: $A_{3,3} \rightarrow L_{3,3}U_{3,3}$ 5: $U_{1,3} = L_{1,1}^{-1}A_{1,3}$ 10: $A_{2,2} \rightarrow L_{2,2}U_{2,2}$ 14: $A_{3,3} \rightarrow L_{3,3}U_{3,3}$

Block Distribution for LU

Notice the significant load imbalance

P ₀	P ₃		P ₆		
T ₁	T ₄]	[[] 5	
P ₁	P ₄		P ₇		
T ₂	T ₆ T	10	Т ₈	T ₁₂	
P ₂	P ₅		P ₈		
T ₃	T ₇ T	, 11	T ₉ T	13 ^T 14	

Block Cyclic Distributions

- Variation of the block distribution scheme
 - Partition an array into many more blocks (i.e. tasks) than the number of available processes.
 - Blocks are assigned to processes in a round-robin manner so that each process gets several non-adjacent blocks.
 - N-1 mapping of tasks to processes
- Used to alleviate the load-imbalance and idling problems.



Block-Cyclic Distribution for Gaussian Elimination

- Active submatrix shrinks as elimination progresses
- Assigning blocks in a block-cyclic fashion
 - Each PEs receives blocks from different parts of the matrix
 - In one batch of mapping, the PE doing the most will most likely receive the least in the next batch



Block-Cyclic Distribution

- A cyclic distribution: a special case with block size = 1
- A block distribution: a special case with block size = *n/p*
 - *n* is the dimension of the matrix and *p* is the #of processes.





Block Partitioning and Random Mapping

Sparse matrix computations

- Load imbalance using block-cyclic partitioning/mapping
 - more non-zero blocks to diagonal processes P0, P5, P10, and P15 than others
 - P12 gets nothing



P_0	P_1	P_2	P_3	P_0	P_1	P_2	P_3
P_4	P_5	P_6	P_7	P_4	P_5	P_6	P_7
P_8	P_9	P_{10}	<i>P</i> ₁₁	P_8	P_9	P_{10}	P ₁₁
P_{12}	<i>P</i> ₁₃	P_{14}	P_{15}	P_{12}	P ₁₃	P_{14}	P ₁₅
P_0	P_1	P_2	P_3	P_0	P_1	P_2	P_3
P_4	P_5	P_6	<i>P</i> ₇	P_4	P_5	P_6	P_7
P_8	P_9	P_{10}	<i>P</i> ₁₁	P_8	P_9	P_{10}	<i>P</i> ₁₁
P_{12}	P ₁₃	P_{14}	P ₁₅	P_{12}	P ₁₃	P_{14}	P ₁₅

(a)

(b)

Block Partitioning and Random Mapping

V = [0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]

random(V) = [8, 2, 6, 0, 3, 7, 11, 1, 9, 5, 4, 10]



(a)

(b)

(c)

Graph Partitioning Based Data Decomposition

- Array-based partitioning and static mapping
 - Regular domain, i.e. rectangular, mostly dense matrix
 - Structured and regular interaction patterns
 - Quite effective in balancing the computations and minimizing the interactions
- Irregular domain
 - Spars matrix-related
 - Numerical simulations of physical phenomena
 - Car, water/blood flow, geographic
- Partition the irregular domain so as to
 - Assign equal number of nodes to each process
 - Minimizing edge count of the partition.

Partitioning the Graph of Lake Superior

- Each mesh point has the same amount of computation
 - Easy for load balancing
- Minimize edges
- Optimal partition is an NP-complete
 - Use heuristics





Random Partitioning



Partitioning for minimum edge-cut.

Mappings Based on Task Paritioning

- Schemes for Static Mapping
 - Mappings based on data partitioning
 - Mostly 1-1 mapping
 - Mappings based on task graph partitioning
 - Hybrid mappings
- Data partitioning
 - Data decomposition and then 1-1 mapping of tasks to PEs

Partitioning a given task-dependency graph across processes

- An optimal mapping for a general task-dependency graph
 NP-complete problem.
- Excellent heuristics exist for structured graphs.

Mapping a Binary Tree Dependency Graph

Mapping dependency graph of quicksort to processes in a hypercube

- Hypercube: n-dimensional analogue of a square and a cube
 - node numbers that differ in 1 bit are adjacent



Mapping a Sparse Graph

Sparse matrix vector multiplication

Using data partitioning



Mapping a Sparse Graph

Sparse matrix vector multiplication

Using task graph partitioning



Hierarchical/Hybrid Mappings

- A single mapping is inadequate.
 - E.g. task graph mapping of the binary tree (quicksort) cannot use a large number of processors.
- Hierarchical mapping
 - Task graph mapping at the top level
 - Data partitioning within each level.



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Schemes for Dynamic Mapping

- Also referred to as dynamic load balancing
 - Load balancing is the primary motivation for dynamic mapping.
- Dynamic mapping schemes can be
 - Centralized
 - Distributed

Centralized Dynamic Mapping

- Processes are designated as masters or slaves
 - Workers (slave is politically incorrect)
- General strategies
 - Master has pool of tasks and as central dispatcher
 - When one runs out of work, it requests from master for more work.
- Challenge
 - When process # increases, master may become the bottleneck.
- Approach
 - Chunk scheduling: a process picks up multiple tasks at once
 - Chunk size:
 - Large chunk sizes may lead to significant load imbalances as well
 - Schemes to gradually decrease chunk size as the computation progresses.



Distributed Dynamic Mapping

- All processes are created equal
 - Each can send or receive work from others
 - Alleviates the bottleneck in centralized schemes.
- Four critical design questions:
 - how are sending and receiving processes paired together
 - who initiates work transfer
 - how much work is transferred
 - when is a transfer triggered?
- Answers are generally application specific.
- Workstealing

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Minimizing Interaction Overheads

Rules of thumb

- Maximize data locality
 - Where possible, reuse intermediate data
 - Restructure computation so that data can be reused in smaller time windows.
- Minimize volume of data exchange
 - partition interaction graph to minimize edge crossings
- Minimize frequency of interactions
 - Merge multiple interactions to one, e.g. aggregate small msgs.
- Minimize contention and hot-spots
 - Use decentralized techniques
 - Replicate data where necessary

Minimizing Interaction Overheads (continued)

Techniques

- Overlapping computations with interactions
 - Use non-blocking communications
 - Multithreading
 - Prefetching to hide latencies.
- Replicating data or computations to reduce communication
- Using group communications instead of point-to-point primitives.
- Overlap interactions with other interactions.

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Parallel Algorithm Models

- Ways of structuring parallel algorithm
 - Decomposition techniques
 - Mapping technique
 - Strategy to minimize interactions.
- Data Parallel Model
 - Each task performs similar operations on different data
 - Tasks are statically (or semi-statically) mapped to processes
- Task Graph Model
 - Use task dependency graph to guide the model for better locality or low interaction costs.

Parallel Algorithm Models (continued)

- Master-Slave Model
 - Master (one or more) generate work
 - Dispatch work to workers.
 - Dispatching may be static or dynamic.
- Pipeline / Producer-Consumer Model
 - Stream of data is passed through a succession of processes, each of which perform some task on it
 - Multiple stream concurrently
- Hybrid Models
 - Applying multiple models hierarchically
 - Applying multiple models sequentially to different phases of a parallel algorithm.

References

- Adapted from slides "Principles of Parallel Algorithm Design" by Ananth Grama
- Based on Chapter 3 of "Introduction to Parallel Computing" by Ananth Grama, Anshul Gupta, George Karypis, and Vipin Kumar. Addison Wesley, 2003