# Friday 11/09 Class and Next Week

- Friday 11/09, 9:00AM, 2A31 (Same classroom)
  - I will post assignment 4 by today
- Next week:
  - Monday: No class
  - Wednesday: regular time via <a href="https://webconnect.sc.edu/csce513/">https://webconnect.sc.edu/csce513/</a>
    - » Check <a href="https://passlab.github.io/CSCE513/OnlineAdobeConnect.html">https://passlab.github.io/CSCE513/OnlineAdobeConnect.html</a> for details how to make your computer ready
- The week after next (11/19 week)
  - Monday 11/19 regular class
  - Wednesday 11/21, No class because of Thanksgiving
- Then there are two more weeks for this course.
- Final Exam: 9:00AM 12/12 Wednesday

# Lecture 20: Data Level Parallelism -- Introduction and Vector Architecture

**CSCE 513 Computer Architecture** 

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https://passlab.github.io/CSCE513

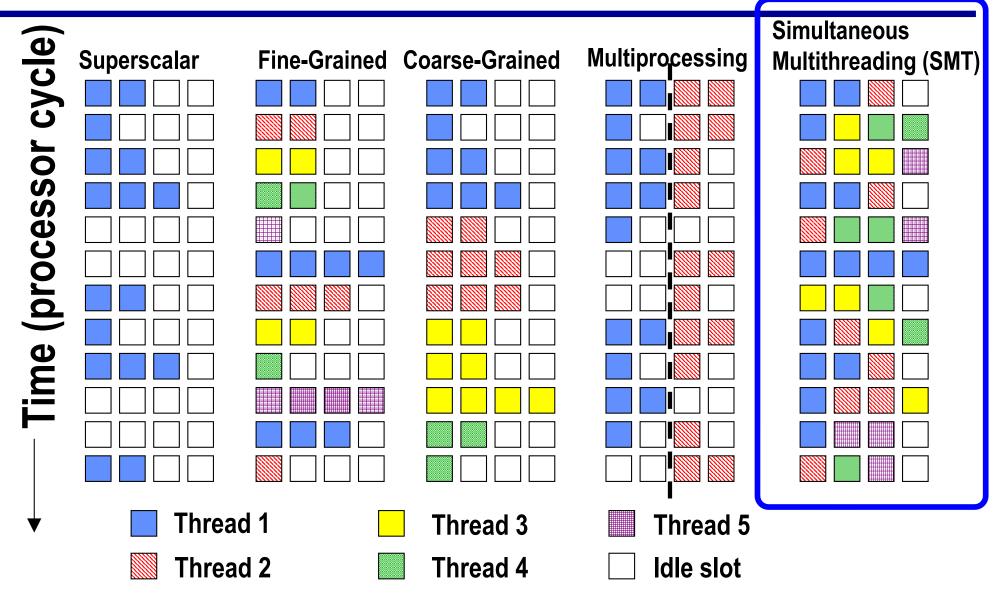
## Very Important Terms for Instruction-Level Parallelism

- Dynamic Scheduling -> Out-of-order Execution
- Speculation → In-order Commit
- Superscalar → Multiple Issue

Techniques	Goals	Implementation	Addressing	Approaches
Dynamic Scheduling	Out-of- order execution	Reservation Stations, Load/Store Buffer and CDB	Data hazards (RAW, WAW, WAR)	Register renaming
Speculation	In-order commit	Branch Prediction (BHT/BTB) and Reorder Buffer	Control hazards (branch, func, exception)	Prediction and misprediction recovery
Superscalar /VLIW	Multiple issue	Software and Hardware	To Increase CPI	By compiler or hardware

Though mostly invented earlier, these techniques are still widely used today, in from embedded CPUs to server/desktop CPUs.

# Multithreading: Hyper-Threading = SMT



**SMT/HTT** performance improvement is **NOT** significant!

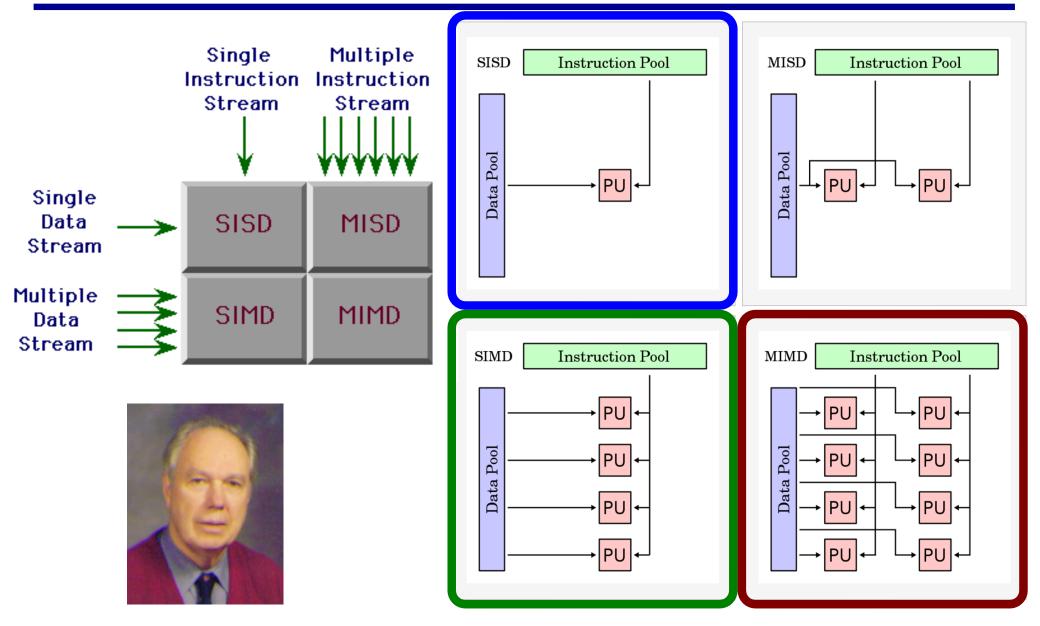
#### **CSE 564 Class Contents**

- Introduction to Computer Architecture (CA)
- Quantitative Analysis, Trend and Performance of CA
  - Chapter 1
- Instruction Set Principles and Examples
  - Appendix A
- Pipelining and Implementation, RISC-V ISA and Implementation
  - Appendix C, RISC-V (riscv.org) and UCB RISC-V impl
- Memory System (Technology, Cache Organization and Optimization, Virtual Memory)
  - Appendix B and Chapter 2
  - Midterm covered till Memory Tech and Cache Organization
- Instruction Level Parallelism (Dynamic Scheduling, Branch Prediction, Hardware Speculation, Superscalar, VLIW and SMT)
  - Chapter 3
- Data Level Parallelism (Vector, SIMD, and GPU)
  - Chapter 4
- Thread Level Parallelism
  - Chapter 5
- Domain-specific architecture
  - Chapter 7

# **Topics for Data Level Parallelism (DLP)**

- Parallelism (centered around ... )
  - Instruction Level Parallelism
  - Data Level Parallelism
  - Thread Level Parallelism
- DLP Introduction and Vector Architecture
  - -4.1, 4.2
- SIMD Instruction Set Extensions for Multimedia
  - -4.3
- Graphical Processing Units (GPU)
  - -4.4
- GPU and Loop-Level Parallelism and Others
  - -4.4, 4.5

# Flynn's Taxonomy for Classifying CA



Michael J. Flynn: http://arith.stanford.edu/~flynn/

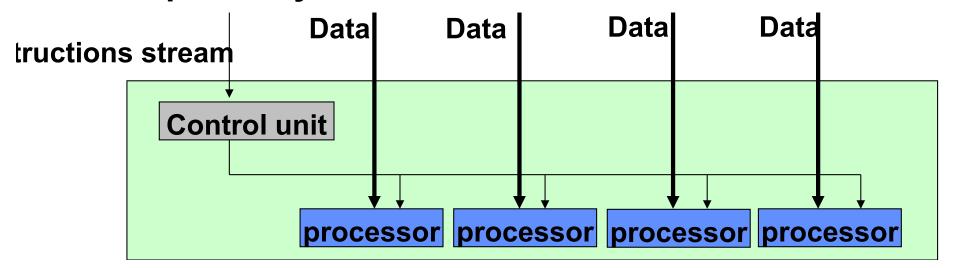
# Flynn's Classification (1966)

#### Broad classification of parallel computing systems

- based upon the number of concurrent Instruction
   (or control) streams and Data streams
- SISD: Single Instruction, Single Data
  - conventional uniprocessor, a single core
- SIMD: Single Instruction, Multiple Data
  - one instruction stream, multiple data paths
  - distributed memory SIMD (MPP, DAP, CM-1&2, Maspar)
  - shared memory SIMD (STARAN, vector computers)
- MIMD: Multiple Instruction, Multiple Data
  - message passing machines (Transputers, nCube, CM-5, clusters)
  - non-cache-coherent shared memory machines (BBN Butterfly, T3D)
  - cache-coherent shared memory machines (Multicore, multiprocessors, Sequent, Sun Starfire, SGI Origin)
- MISD: Multiple Instruction, Single Data
  - Not a practical configuration

# SIMD: Single Instruction, Multiple Data (Data Level Parallelism)

- SIMD architectures can exploit significant datalevel parallelism for:
   for (i=999; i>=0; i=i-1)
  - matrix-oriented scientific computing
  - media-oriented image and sound processors
- SIMD is more energy efficient than MIMD
  - Only needs to fetch one instruction per data operation processing multiple data elements
  - Makes SIMD attractive for personal mobile devices
- SIMD allows programmer to continue to think sequentially



x[i] = x[i] + s;

# Hardware Implementation for SIMD|Data-Level Parallelism

- Three variations
  - Vector architectures
  - SIMD extensions
  - Graphics Processor Units (GPUs)
- E.g. x86 processors → MIMD + SIMD
  - Expect two additional cores per chip per year (MIMD)
  - Each core has SIMD, and SIMD width double every four years
  - Potential speedup from SIMD to be twice that from MIMD!

#### **Vector Architecture**

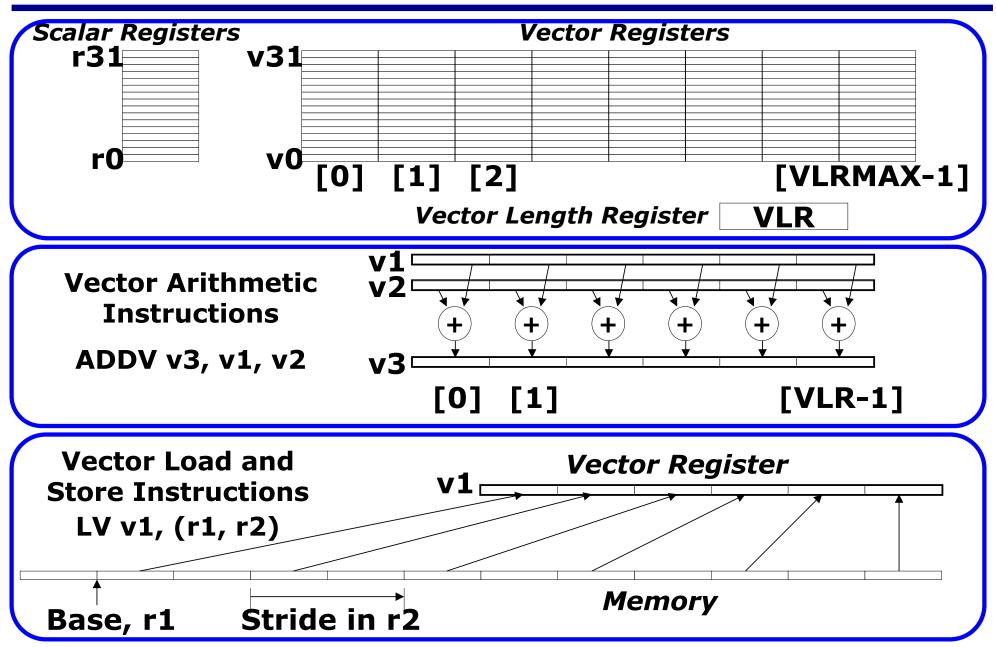
#### **VLIW** vs Vector

 VLIW takes advantage of instruction level parallelism (ILP) by specifying multiple (different) instructions to execute in parallel

Int Op 1 Int Op 2 Mem Op	. Mem Op 2 FP Op 1	FP Op 2
--------------------------	--------------------	---------

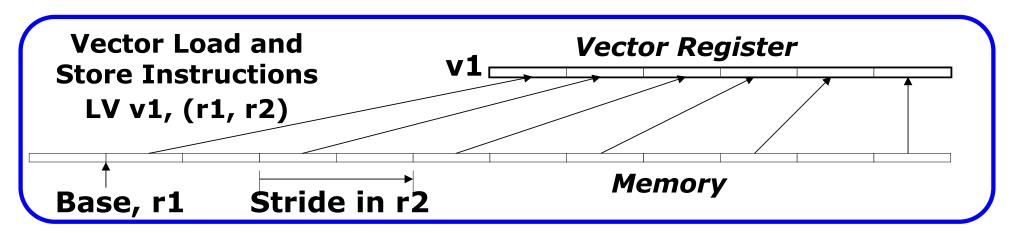
- Vector architectures perform the same operation on multiple data elements – single instruction
  - Data-level parallelism

# **Vector Programming Model**



#### **Control Information**

- Vector length (VL) register limits the max number of elements to be processed by a vector instruction
  - VL is loaded prior to executing the vector instruction with a special instruction
- Stride for load/stores:
  - Vectors may not be adjacent in memory addresses
  - E.g., different dimensions of a matrix
  - Stride can be specified as part of the load/store



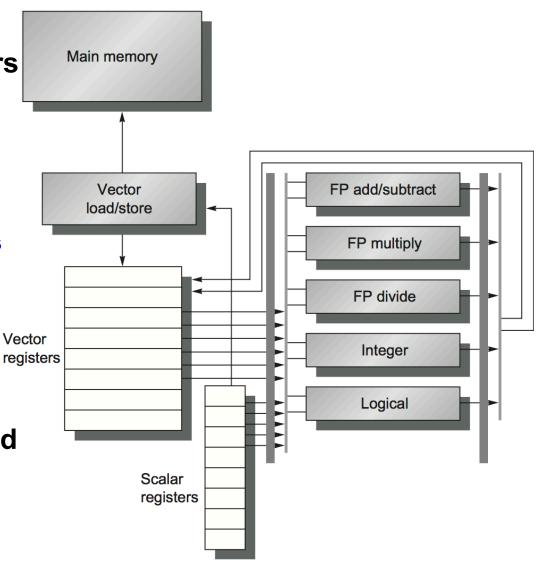
#### **Basic Structure of Vector Architecuture**

RV64V

32 32x8-byte vector registers

 all the functional units are vector functional units.

- To sustain requests of high memory bandwidth
  - The vector and scalar registers have a significant number of read and write ports to allow multiple simultaneous vector operations.
- A set of crossbar switches (thick gray lines) connects these ports to the inputs and outputs of the vector functional units.



**Figure 4.1** The basic structure of a vector architecture, RV64V, which includes RISC-V scalar architecture. There are also 32 vector registers, and all the functional ur are vector functional units. The vector and scalar registers have a significant number read and write ports to allow multiple simultaneous vector operations. A set of crossless

#### **RV64V Vector Instructions**

#### Suffix

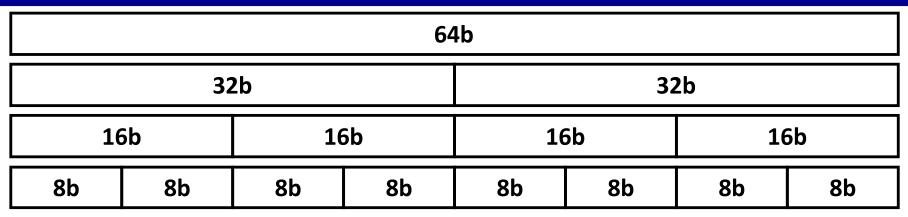
- V suffix
- VS suffix
- Load/Store
  - vld
  - vst
- Registers
  - V registers
  - VL (vector length register)
  - Predicate

Mnemonic	Name	Description	
vadd	ADD	Add elements of V[rs1] and V[rs2], then put each result in V[rd]	
vsub	SUBtract	Subtract elements of V[rs2] frpm V[rs1], then put each result in V[rd]	
vmul	MULtiply	Multiply elements of V[rs1] and V[rs2], then put each result in V[rd]	
vdiv	DIVide	Divide elements of V[rs1] by V[rs2], then put each result in V[rd]	
vrem	REMainder Take remainder of elements of V[rs1] by V[rs2], then put each result in V[rd]		
vsqrt	SQuare RooT	Take square root of elements of V[rs1], then put each result in V[rd]	
vsll	Shift Left	Shift elements of V[rs1] left by V[rs2], then put each result in V[rd]	
vsrl	Shift Right	Shift elements of V[rs1] right by V[rs2], then put each result in V[rd]	
vsra	Shift Right Arithmetic	Shift elements of $V[rs1]$ right by $V[rs2]$ while extending sign bit, then put each result in $V[rd]$	
vxor	XOR	Exclusive OR elements of V[rs1] and V[rs2], then put each result in V[rd]	
vor	OR	Inclusive OR elements of V[rs1] and V[rs2], then put each result in V[rd]	
vand	AND	Logical AND elements of V[rs1] and V[rs2], then put each result in V[rd]	
vsgnj	SiGN source	Replace sign bits of V[rs1] with sign bits of V[rs2], then put each result in V[rd]	
vsgnjn	Negative SiGN source	Replace sign bits of $V[rs1]$ with complemented sign bits of $V[rs2]$ , then put each result in $V[rd]$	
vsgnjx	Xor SiGN source	Replace sign bits of V[rs1] with xor of sign bits of V[rs1] and V[rs2], then put each result in V[rd]	
vld	Load	Load vector register V[rd] from memory starting at address R[rs1]	
vlds	Strided Load	Load V[rd] from address at R[rs1] with stride in R[rs2] (i.e., R[rs1]+i×R[rs2])	
vldx	Indexed Load (Gather)	Load V[rs1] with vector whose elements are at R[rs2]+V[rs2] (i.e., V[rs2] is an index)	
vst	Store	Store vector register V[rd] into memory starting at address R[rs1]	
vsts	Strided Store	Store V[rd] into memory at address R[rs1] with stride in R[rs2] (i.e., R[rs1] + $i \times R[rs2]$ )	
vstx	Indexed Store (Scatter)	Store $V[rs1]$ into memory vector whose elements are at $R[rs2] + V[rs2]$ ( i.e., $V[rs2]$ is an index)	
vpeq	Compare =	ripare = Compare elements of V[rs1] and V[rs2]. When equal, put a 1 in the corresponding 1-bit element of p[rd]; otherwise, put 0	
vpne	Compare !=	Compare elements of V[rs1] and V[rs2]. When not equal, put a 1 in the corresponding 1-bit element of p[rd]; otherwise, put 0	
vplt	Compare <	Compare elements of V[rs1] and V[rs2]. When less than, put a 1 in the corresponding 1-	
vpxor	Predicate XOR	Exclusive OR 1-bit elements of p[rs1] and p[rs2], then put each result in p[rd]	
vpor	Predicate OR	Inclusive OR 1-bit elements of p[rs1] and p[rs2], then put each result in p[rd]	
vpand	Predicate AND	Logical AND 1-bit elements of p[rs1] and p[rs2], then put each result in p[rd]	
setvl	Set Vector Length	Set vl and the destination register to the smaller of mvl and the source regsiter	

# Highlight of RV64V Vector Instructions

- All are R-format instruction
- .vv and .vs|.sv operands
  - .vv: Vector-vector operands; .vs|.sv: Vector-scalar operands
- Vector load and store which loads or stores an entire vector
  - One operand is the vector register to be loaded or stored; The other operand, a GPR, is the starting address of the vector in memory.
  - vlds/vsts: for stride load/store; vldx/vstx: indexed load/store
- Vector-length register vI is used when the natural vector length is not equal to mvI
- Vector-type register vctype records register types
- Predicate registers pi are used when loops involve IF statements.
- We'll see them in the following example:

# **Dynamic Register Typing in RV64V**



- A vector register has 32 64-bit elements
  - Or 128 16- bit elements, and even 256 8-bit elements are equally valid views.

T .	0 16 00 16414	TT	16 22 16414
Integer	8, 16, 32, and 64 bits	Floating point	16, 32, and 64 bits

Figure 4.3 Data sizes supported for RV64V assuming it also has the single- and double-precision floating-point extensions RVS and RVD. Adding RVV to such a

- Associate a data type and data size with each vector register using vctype register
  - Existing and normal approach is that the instruction supplying the type information, but not in RV64V

# DAXPY(Y = a \* X + Y, 32 elements) in RV64G and RV64V

```
double a, X[], Y[]; // 8-byte per element
for (i=0; i<32; i++)
  Y[i] = a* X[i] + Y[i];</pre>
```

The starting addresses of X and Y are in X5 and X6, respectively

- # instrs:
  - 8 vs 258
- Pipeline stalls
  - 32x higher by RV64G
- Vector chaining (forwarding)
  - Per each vector element
  - v0→v1→v2→v3

```
fld
             f0,a
                             # Load scalar a
      addi
             x28, x5, #256
                             # Last address to load
Loop: fld f1,0(x5)
                             # Load X[i]
      fmul.d f1,f1,f0
                             \# a \times X[i]
             f2,0(x6)
      fld
                             # Load Y[i]
      fadd.d f2, f2, f1
                             \# a \times X[i] + Y[i]
      fsd f2,0(x6)
                             # Store into Y[i]
      addi x5,x5,#8
                             # Increment index to X
            x6,x6,#8
      addi
                             # Increment index to Y
      bne
             x28,x5,Loop
                             # Check if done
```

```
vsetdcfg
          4*FP64
                          # Enable 4 DP FP vregs
fld
          f0.a
                          # Load scalar a
bſv
          v0.x5
                          # Load vector X
vmu1
          v1,v0,f0
                          # Vector-scalar mult
                          # Load vector Y
vld
          v2,x6
          v3,v1,v2
                          # Vector-vector add
vadd
          v3.x6
                          # Store the sum
vst
vdisable
                          # Disable vector regs
```

# DAXPY(Y = a \* X + Y, 32 elements) in RV64G and RV64V

fO.a

v0.x5

v1,v0,f0

```
double a, X[], Y[]; // 8-byte per element
for (i=0; i<32; i++)
   Y[i] = a* X[i] + Y[i];
   vsetdcfg 4*FP64</pre>
```

Y[i] = a\* X[i] + Y[i];

The starting addresses of X and Y are in X5 and X6, respectively

# Load scalar a

# Load vector X

# Vector-scalar mult

# Enable 4 DP FP vregs

```
vld v2,x6 # Load vector Y
vadd v3,v1,v2 # Vector-vector add
vst v3,x6 # Store the sum
vdisable # Disable vector regs

for (i=0; i<32; i++)
```

vmul

```
vsetdcfg 1*FP32,3*FP64 # 1 32b, 3 64b vregs
flw f0,a # Load scalar a
vld v0,x5 # Load vector X
vmul v1,v0,f0 # Vector-scalar mult
vld v2,x6 # Load vector Y
vadd v3,v1,v2 # Vector-vector add
vst v3,x6 # Store the sum
vdisable # Disable vector regs
```

# DAXPY(Y = a \* X + Y, 32 elements) in RV64G and RV64V

```
double a, X[], Y[]; // 8-byte per element
                                            The starting addresses of X and
for (i=0; i<32; i++)
                                            Y are in X5 and X6, respectively
  Y[i] = a* X[i] + Y[i];
                          vsetdcfg
                                   4*FP64
                                                 # Enable 4 DP FP vregs
                          fld
                                   f0,a
                                                 # Load scalar a
                          vld
                                  v0.x5
                                                 # Load vector X
                                               # Vector-scalar mult
                                  v1,v0,f0
                          vmul
                          vld
                                 v2,x6
                                                 # Load vector Y
                          vadd v3,v1,v2 # Vector-vector add
                          vst
                                  v3,x6
                                                # Store the sum
                                                 # Disable vector regs
                          vdisable
int a, X[], Y[];
                          vsetdcfg 1*X32,3*X64 # 1 32b, 3 64b int reg
for (i=0; i<32; i++)
                                               # Load scalar a
                          ٦w
                                   x7,a
  Y[i] = a* X[i] + Y[i];
                                  v0,x5
                          v]d
                                               # Load vector X
                                v1,v0,x7 # Vector-scalar mult
                          vmul
                                 v2,x6
                                               # Load vector Y
                          vld
                          vadd v3,v1,v2
                                               # Vector-vector add
                                v3,x6
                                               # Store the sum
                          vst
                          vdisable
                                               # Disable vector regs
```

## Vector Instruction Set Advantages

#### Compact

- one short instruction encodes N operations
- Expressive and predictable, tells hardware that these N operations:
  - are independent
  - use the same functional unit
  - access disjoint registers
  - access registers in same pattern as previous instructions
  - access a contiguous block of memory (unit-stride load/store)
  - access memory in a known pattern (strided load/store)

#### Scalable

can run same code on more parallel pipelines (lanes)

# **Vector Length Register**

Loop count not known at compile time?

```
for (i=0; i <n; i=i+1)
Y[i] = a * X[i] + Y[i];
```

- Use Vector Length (VL) and Max VL (MVL) Registers
- Use strip mining for vectors over the maximum length (serialized version before vectorization by compiler)
  - Break loops into pieces that fit in registers

```
Iow = 0;

VL = (n % MVL); /*find odd-size piece using modulo op % */

for (j = 0; j <= (n/MVL); j=j+1) { /*outer loop*/

for (i = low; i < (low+VL); i=i+1) /*runs for length VL*/

Y[i] = a * X[i] + Y[i]; /*main operation*/

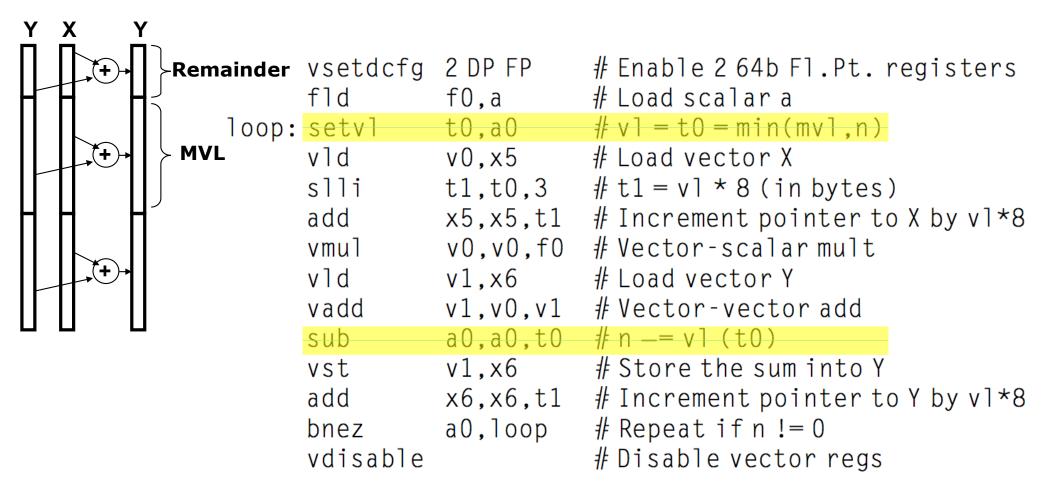
Iow = low + VL; /*start of next vector*/

VL = MVL; /*reset the length to maximum vector length*/

}
```

### **Vector Stripmining in RV64V**

```
for (i=0; i <n; i=i+1)
Y[i] = a * X[i] + Y[i];</pre>
```



# **Using Predictate Register for Vector Mask**

```
for (i = 0; i < 64; i=i+1)
if (X[i]!=0)
X[i] = X[i] - Y[i];</pre>
```

# Use predicate register to "disable" elements (1 bit per element):

```
vsetdcfg
vsetpcfgi
vld
vld
vld
vl,x6
fmv.d.x

vpne
vsub
vst
vdisable
vpdisable
vsetpcfgi
1
# Enable 2 64b FP vector regs
# Enable 1 predicate register
# Load vector X into v0
# Load vector Y into v1
# Load vector Y into v1
# Put (FP) zero into f0

# Set p0(i) to 1 if v0(i)!=f0
# Subtract under vector mask
# Store the result in X
# Disable vector registers
# Disable predicate registers
```

#### GFLOPS rate decreases!

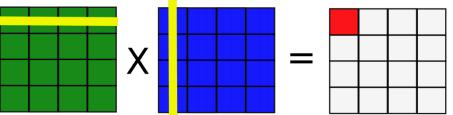
 Vector operation becomes bubble ("NOP") at elements where mask bit is clear

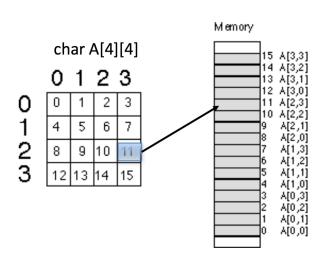
#### **Stride**

# DGEMM (Double-Precision Matrix Multiplication)



- Row-major: B: 1 double (8 bytes), and D: 100 doubles (800 bytes)
- Use non-unit stride
  - vlds and vsts: strided load and store
- Bank conflict (stall) occurs when the same bank is hit faster than bank busy time:
  - #banks / LCM(stride,#banks) < bank busy time





#### **Scatter-Gather**

#### Sparse matrix:

- Non-zero values are compacted to a smaller value array (A[])
- indirect array indexing, i.e. use an array to store the index to value array (K[])

```
for (i = 0; i < n; i=i+1)

A[K[i]] = A[K[i]] + C[M[i]];
```

```
\begin{pmatrix} 1.0 & 0 & 5.0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3.0 & 0 & 0 & 0 & 0 & 11.0 & 0 \\ 0 & 0 & 0 & 0 & 9.0 & 0 & 0 & 0 \\ 0 & 0 & 6.0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 7.0 & 0 & 0 & 0 & 0 \\ 2.0 & 0 & 0 & 0 & 0 & 10.0 & 0 & 0 \\ 0 & 0 & 0 & 8.0 & 0 & 0 & 0 & 0 \\ 0 & 4.0 & 0 & 0 & 0 & 0 & 0 & 12.0 \end{pmatrix}
```

#### Use index vector:

```
# 4 64b FP vector registers
vsetdcfg
        4*FP64
      v0, x7 # Load K[]
vld
vldx
      v1,(x5,v0) # Load A[K[]]
      v2, x28
               # Load M[]
v1d
vldi v3,(x6, v2) # Load C[M[]]
    v1, v1, v3 # Add them
vadd
    v1,(x5, v0) # Store A[K[]]
vstx
                    # Disable vector registers
vdisable
```

# **Memory Operations (vld and vst)**

- Load/store operations move groups of data between registers and memory
  - Increased mem/instr ratio (intensity)
- Three types of addressing
  - Unit stride
    - » Contiguous block of information in memory
    - » Fastest: always possible to optimize this
  - Non-unit (constant) stride
    - » Harder to optimize memory system for all possible strides
    - » Prime number of data banks makes it easier to support different strides at full bandwidth
  - Indexed (gather-scatter)
    - » Vector equivalent of register indirect
    - » Good for sparse arrays of data
    - » Increases number of programs that vectorize

#### Conclusion

#### Vector is alternative model for exploiting ILP

- If code is vectorizable, then simpler hardware, more energy efficient, and better real-time model than Out-of-order machines
- Design issues include number of lanes, number of functional units, number of vector registers, length of vector registers, exception handling, conditional operations

# History: In 70s-80s, Supercomputer ≡ Vector Machine

- Definition of a supercomputer:
  - Fastest machine in world at given task
  - A device to turn a compute-bound problem into an I/O bound problem
  - Any machine costing \$30M+
  - Any machine designed by Seymour Cray (originally)
- CDC6600 (Cray, 1964) regarded as first supercomputer
  - A vector machine
  - <u>www.cray.com</u>: The Supercomputer Company
- Today's supercomputer
  - https://www.top500.org/

#### The Father of Supercomputing



#### Seymour Cray

<

Electrical engineer

Seymour Roger Cray was an American electrical engineer and supercomputer architect who designed a series of computers that were the fastest in the world for decades, and founded Cray Research which built many of these machines. Wikipedia

**Born:** September 28, 1925, Chippewa Falls, WI **Died:** October 5, 1996, Colorado Springs, CO

Awards: Eckert-Mauchly Award

Parents: Seymour R. Cray, Lillian Cray

Education: University of Minnesota, Chippewa Falls High School

Fields: Applied mathematics, Computer Science, Electrical engineering

https://en.wikipedia.org/wiki/Seymour Cray

http://www.cray.com/company/history/seymour-cray

## Supercomputer Applications

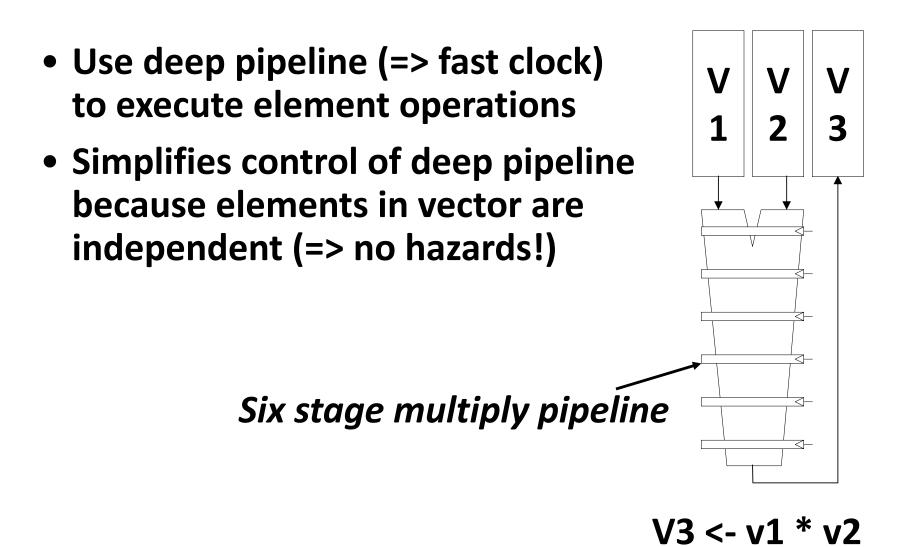
- Typical application areas
  - Military research (nuclear weapons, cryptography)
  - Scientific research
  - Weather forecasting
  - Oil exploration
  - Industrial design (car crash simulation)
  - Bioinformatics
  - Cryptography
- All involve huge computations on large data sets

# **Vector Supercomputers**

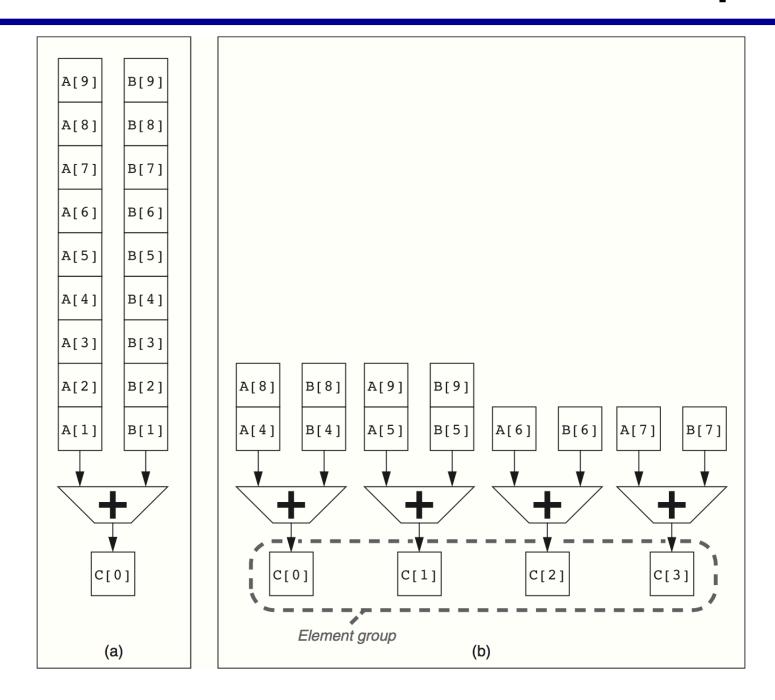
- Epitomy: Cray-1, 1976
- **Scalar Unit** 
  - Load/Store Architecture
- Vector Extension
  - Vector Registers
  - Vector Instructions
- **Implementation** 
  - Hardwired Control
  - Highly Pipelined Functional Units
  - Interleaved Memory System
  - No Data Caches
  - No Virtual Memory



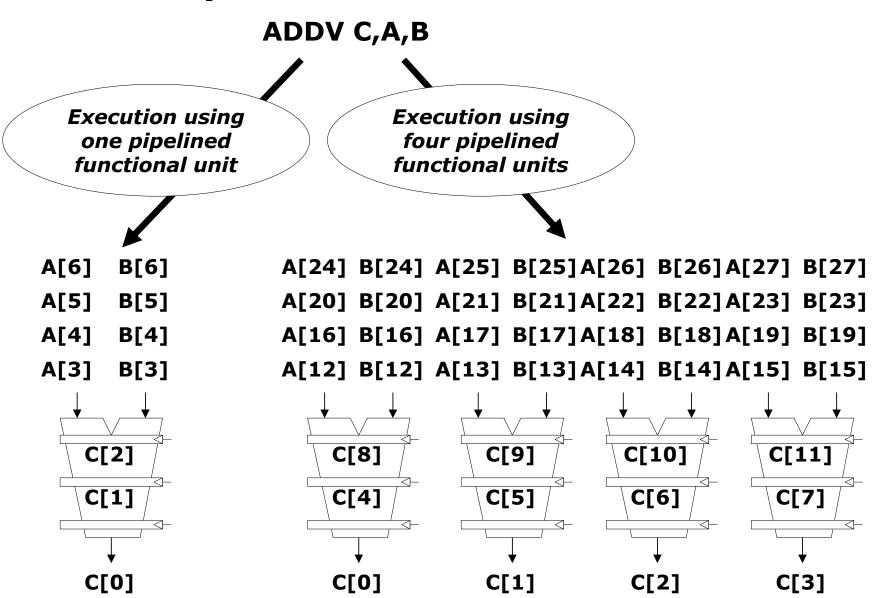
#### **Vector Arithmetic Execution**



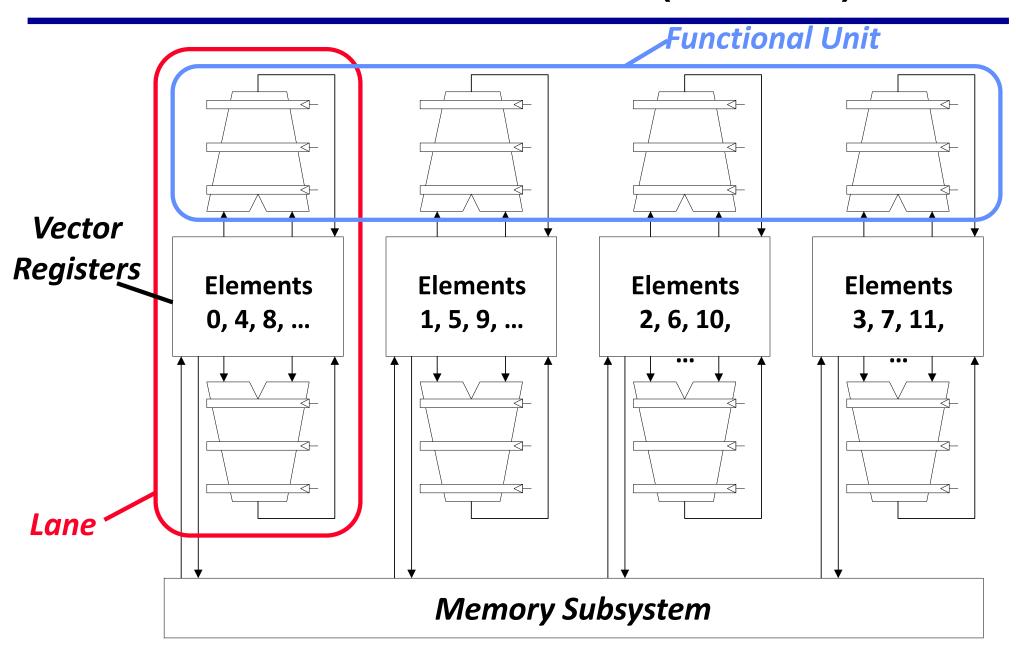
## **Vector Execution: Element Group**



# Vector Instruction Execution with Pipelined Functional Units

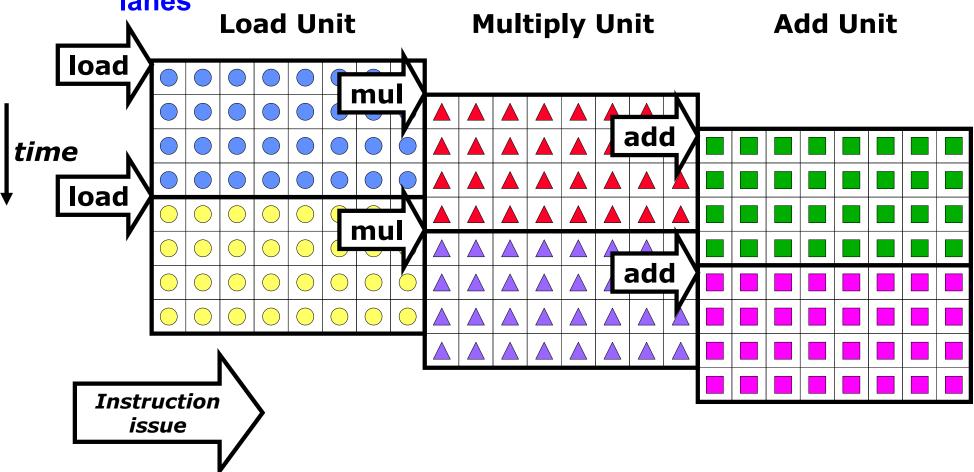


# **Vector Unit Structure (4 Lanes)**



#### Vector Instruction Parallelism

- Can overlap execution of multiple vector instructions
  - example machine has 32 elements per vector register and 8 lanes

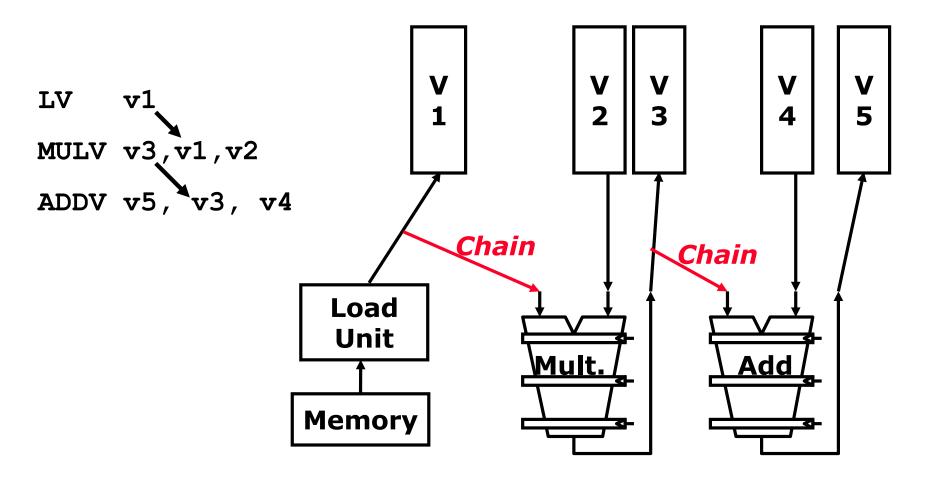


Complete 24 operations/cycle while issuing 1 short instruction/cycle

## **Vector Chaining**

#### Vector version of register bypassing

- introduced with Cray-1

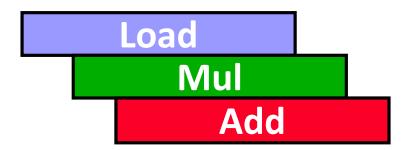


# **Vector Chaining Advantage**

 Without chaining, must wait for last element of result to be written before starting dependent instruction



 With chaining, can start dependent instruction as soon as first result appears

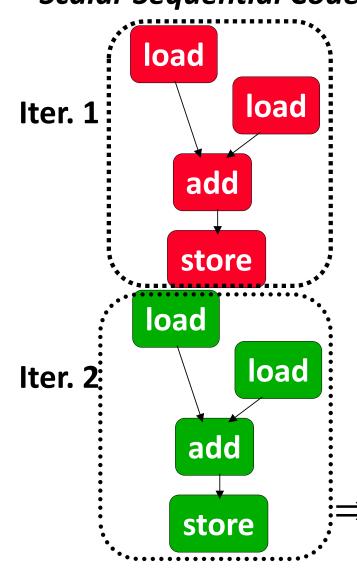


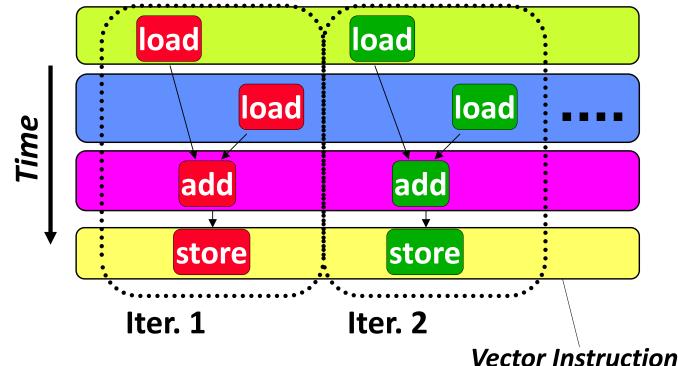
### **Class Lecture Ends Here!**

#### **Automatic Code Vectorization**

for (i=0; i < N; i++)

Scalar Sequential Code C[i] = A[i] + B[i]; Vectorized Code



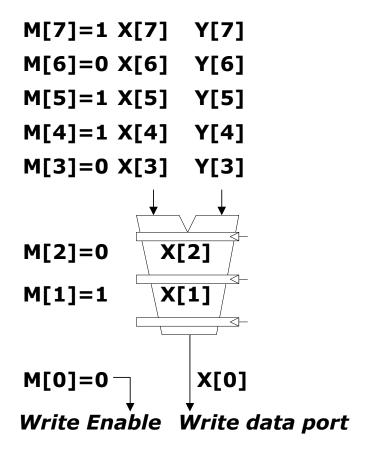


Vectorization is a massive compile-time reordering of operation sequencing requires extensive loop dependence analysis

#### **Masked Vector Instructions**

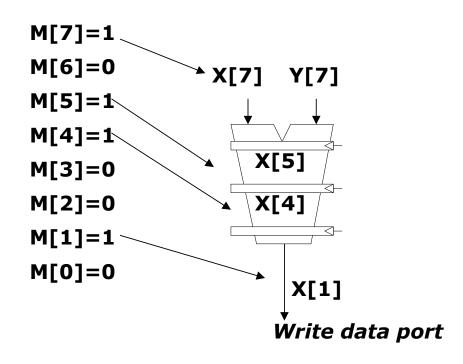
#### **Simple Implementation**

execute all N operations, turn off result writeback according to mask



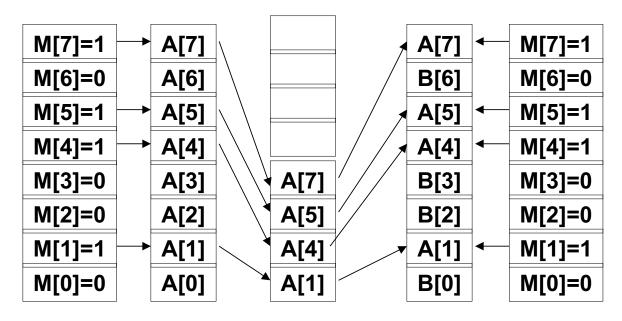
#### **Density-Time Implementation**

 scan mask vector and only execute elements with non-zero masks



# **Compress/Expand Operations**

- Compress packs non-masked elements from one vector register contiguously at start of destination vector register
  - population count of mask vector gives packed vector length
- Expand performs inverse operation

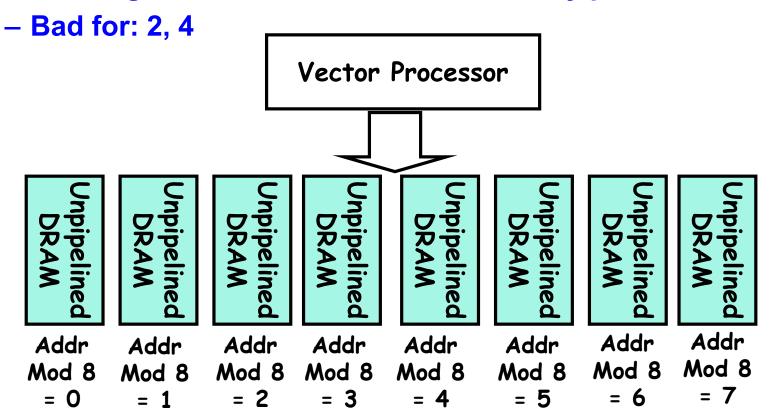


Compress Expand

Used for density-time conditionals and also for general selection operations

# **Interleaved Memory Layout**

- Great for unit stride:
  - Contiguous elements in different DRAMs
  - Startup time for vector operation is latency of single read
- What about non-unit stride?
  - Above good for strides that are relatively prime to 8



# **Avoiding Bank Conflicts**

Lots of banks

```
int x[256][512];
  for (j = 0; j < 512; j = j+1)
     for (i = 0; i < 256; i = i+1)
     x[i][j] = 2 * x[i][j];</pre>
```

- Even with 128 banks, since 512 is multiple of 128, conflict on word accesses
- SW: loop interchange or declaring array not power of 2 ("array padding")
- HW: Prime number of banks
  - bank number = address mod number of banks
  - address within bank = address / number of words in bank
  - modulo & divide per memory access with prime no. banks?
  - address within bank = address mod number words in bank
  - bank number? easy if 2<sup>N</sup> words per bank

# Finding Bank Number and Address within a bank

- Problem: Determine the number of banks, N<sub>b</sub> and the number of words in each bank, N<sub>w</sub>, such that:
  - given address x, it is easy to find the bank where x will be found, B(x), and the address of x within the bank, A(x).
  - for any address x, B(x) and A(x) are unique
  - the number of bank conflicts is minimized
- Solution: Use the Chinese remainder theorem to determine B(x) and A(x):

```
B(x) = x \text{ MOD } N_b

A(x) = x \text{ MOD } N_w where N_b and N_w are co-prime (no factors)
```

- Chinese Remainder Theorem shows that B(x) and A(x) unique.
- Condition allows N<sub>w</sub> to be power of two (typical) if N<sub>b</sub> is prime of form 2<sup>m</sup>-1.
- Simple (fast) circuit to compute (x mod N<sub>b</sub>) when N<sub>b</sub> = 2<sup>m</sup>-1:
  - Since  $2^k = 2^{k-m} (2^m-1) + 2^{k-m} \Rightarrow 2^k \text{ MOD } N_b = 2^{k-m} \text{ MOD } N_b = ... = 2^j \text{ with } j < m$
  - And, remember that: (A+B) MOD C = [(A MOD C)+(B MOD C)] MOD C
  - for every power of 2, compute single bit MOD (in advance)
  - B(x) = sum of these values MOD N<sub>b</sub>
     (low complexity circuit, adder with ~ m bits)